## Abstract

Modern astrophysics relies heavily on numerical simulations to interpret observational data, test theoretical models, understand complex physical processes, and make predictions about future events. Solar and stellar physics are no exception. This thesis deals with numerical simulations of solar and stellar magneto-convection, with the overall aim of presenting new tools, models, and insights to improve our understanding of magneto-hydrodynamic (MHD) processes in stellar interiors and in the solar atmosphere.

This thesis is divided into two main projects. The first project focuses on the adaptation of the RAMSES code to enable the simulation of low-Mach number, highly turbulent, MHD flows commonly found in stellar interiors. The aim of this project is to develop a three-dimensional, fully compressible, MHD numerical code capable of qualitatively reproducing the results of previous studies of stellar convection, dynamo action, and wave propagation. This would allow the study of the complex interplay between magnetic fields and acoustic oscillations in stellar interiors, as this phenomenon can only be effectively investigated with a compressible code such as RAMSES.

For this purpose, a well-balanced scheme is implemented in the numerical solvers of the RAMSES code. This implementation allows to perform a resolution study of the key properties of the three-dimensional, subsonic, turbulent, MHD plasma selfconsistently emerging from a plane-parallel model of a stellar convection zone. The results of this study effectively demonstrate the validity of the approach and that magnetic fields affect the propagation of acoustic oscillations within stellar interiors.

The second project focuses on the study of small-scale swirling motions observed in the quiet solar atmosphere. Due to their close relationship with the Sun's surface magnetic field, they have the potential to serve as a viable mechanism for the upward transport of energy toward the solar chromosphere and corona. Therefore, the aim of this project is to investigate the nature, dynamics, and properties of these smallscale vortices in three-dimensional, radiative-MHD, numerical simulations of the solar atmosphere, performed with the CO5BOLD code.

Given the complexity and dynamical nature of the flows in the surface layers of the convection zone and in the solar atmosphere, advanced methods are essential to extract accurate information about the swirls from simulation data. To address this, we apply state-of-the-art mathematical criteria to the study of vortices, and we introduce novel tools for the automated identification of vortices and the analysis of their dynamics within numerical simulations. Using these innovative tools, we successfully confirm that the behavior of small-scale vortices is primarily governed by the solar surface magnetic field, and we find strong indications that they represent the plasma response to the propagation of torsional Alfvénic pulses. We also present updated statistical properties of these features, which can be further validated by observations from high-resolution solar telescopes such as DKIST.