Abstract

Numerical simulations are at the core of computational science and engineering, because they allow the study of physical processes without the necessity to run costly experiments. In the field of Astrophysics, they are particularly important for the advancing of the science, as some questions can only be addressed through numerical experiments. Often, hyperbolic systems of partial differential equations are used to model gas dynamics in the universe, subject to strong interacting discontinuities, shocks, rarefying waves and peculiar structures.

The development of high order numerical methods which solve hyperbolic equations systems in such severe conditions is an active research topic. In the thesis, we develop high order numerical methods which are structure preserving. This is interesting because typically structure preserving methods perform better in the long-time stability, which is useful for astrophysical type problems.

The first part of this thesis is motivated by the early stages of planet formation, which can be modeled as a quasi-steady state gas disc orbiting around a central star. When long term evolution is desired, this setup is generally hard to evolve numerically. We present a method that solves the unsteady compressible Euler system with gravity type source terms which can capture steady states (well balanced property) beyond hydrostatic equilibrium on Cartesian coordinates, based on the Runge-Kutta discontinuous Galerkin framework [37]. We test this method in many benchmarks, including the set-up of a protoplanetary disc [154], rotating at a sub-Keplerian velocity, with an embedded planet and show that long-time stability is achieved.

In the second part of this thesis we are motivated by the growing interest in high-order magneto-hydrodynamics (MHD) solvers in Astrophysics. The *structure* to be preserved in MHD equations is the divergence free evolution of the magnetic field. To this end, we studied two classes of methods that solve the linear induction equation (a simplification of MHD): one that preserves the evolution of the divergence of a magnetic field in time approximately, and one which preserves exactly the divergence of a magnetic field in time, through a modified spectral differences method [95].

In the third part of this manuscript, we present some work towards structure preserving learning algorithms. With the increase of data-driven methods being used both in the numerical mathematics community and scientific community, namely, in Astrophysics, the understanding of how to include domain knowledge into data-driven methods is relevant. We build a data-driven limiter which is symmetry and scale invariant, in the spirit of [122], and a show methodology to transfer the data-driven limiter across different numerical schemes. In particular, we developed a limiter for the well known discontinuous Galerkin framework in Cartesian meshes and we show that the same limiter, exposed to training data from a residual distribution method, allows the limiter to be used in a residual distribution method [2], in both Cartesian and unstructured triangular meshes.

Finally, we finish the thesis showing two applications of the developed work in Astrophysics. In the first section, we study the relevancy of high order methods for the simulation of planet-disc interactions and we show that a high-order Cartesian mesh code can achieve very low viscosity compared to a code which is tailor-made for the cylindrical geometry, typically assumed for protoplanetary discs. In the second section, one we study the performance of data-driven methods to emulate pairwise collisions of planetary sized bodies compared to classical (semi-)analytic collision models and the impact of structure preserving learning algorithms in this task. We show that although data-driven methods outperform classical (semi-)analytic collision models, the physics of the problem is not guaranteed to be respected and present some solutions towards physics preserving data-driven algorithms.