The Galaxy Bispectrum as a Probe of Primordial non-Gaussiannity

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Motivation to Study LSS Effects of non-Gaussianity

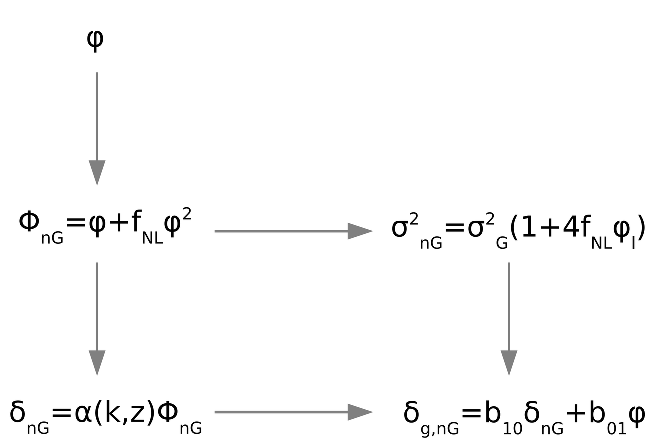
- Goals
  - Learn about the dynamics of the inflaton field
  - LSS provides constraints independent of and complementary to CMB constraints
- Approach
  - Use the bispectrum as a measure of interactions between short and long modes
  - Use perturbation theory to derive shape and scale dependence of the galaxy bispectrum
  - Consistent treatment of non-linear clustering and biased tracers

Imprints of primordial non-Gaussianity on LSS

- An auxiliary Gaussian potential φ shows quadratic and cubic interactions

$$\Phi_{\rm nG}(\boldsymbol{x}) = \varphi(\boldsymbol{x}) + f_{\rm NL}\left(\varphi^2(\boldsymbol{x}) - \left\langle\varphi^2\right\rangle\right) + g_{\rm NL}\varphi^3(\boldsymbol{x})$$

- Effect on the Matter Distribution
  - The nG initial conditions are directly propagated to the final matter distribution which is affected at the percent level at high k
- Effect on the Halo/Galaxy Distribution
  - The variance of short wavelength fluctuations is modulated by long modes $\sigma(M) \to \sigma(M, \varphi_l)$
  - Dark matter haloes collapse once their significance $\nu = \delta_c^2/\sigma^2(M, \varphi_l)$ exceeds unity
  - This leads to an explicit dependence of the halo distribution on φ

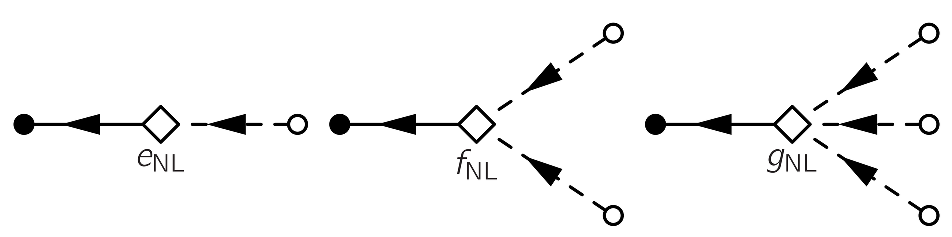


Treatment in Perturbation Theory - Expansion in φ

- Local non-Gaussianity
  - The primordial potential and the linearly evolved primordial matter density field are related by the Poisson factor $\alpha(k) \propto 1/k^2$.
  - The quadratic and cubic terms in the primordial non-Gaussian potential lead to convolutions in Fourier space.

$$\delta(\boldsymbol{k}) = \alpha(k)\varphi(\boldsymbol{k}) + \alpha(k) f_{\rm NL}\int {\rm d}^3q_1\int {\rm d}^3q_2\,\delta^{(\rm D)}(\boldsymbol{k}-\boldsymbol{q})\varphi(\boldsymbol{q}_1)\varphi(\boldsymbol{q}_2) + ...$$

  - These convolutions can be interpreted as interactions between primordial potential modes, which lead to higher order corrections to the density field.



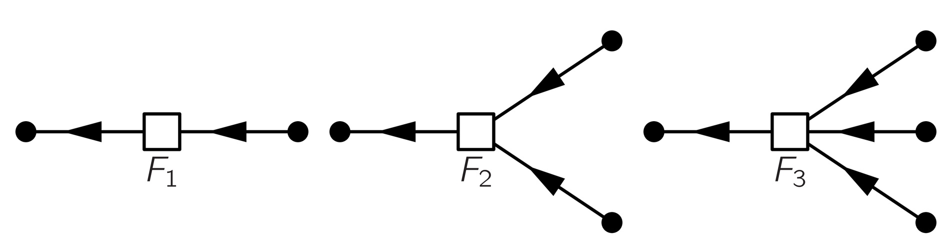
- Non-linear Clustering
  - Following the density perturbations over time requires solving the fluid equations

$$\frac{\partial\delta(\boldsymbol{x},t)}{\partial t} + \theta(\boldsymbol{x},t) = 0 \qquad \nabla^2\Phi = 4\pi G a^2\delta^{(\rm s,c)}$$

$$\frac{\partial\theta(\boldsymbol{x},t)}{\partial\tau} + 2H(t)\theta(\boldsymbol{x},t) + \frac{3}{2}\Omega_{\rm m}H^2(t)\delta(\boldsymbol{x},t) = 0$$

  - Since a closed form solution is not known, the equations are solved perturbatively order by order. The n-th order density field can then be expressed in terms of the linear density fields using the F_n kernels

$$\delta^{(n)}(\boldsymbol{k}) = \int {\rm d}^3q_1 \ldots \int {\rm d}^3q_n\,\delta^{(\rm D)}(\boldsymbol{k}-\boldsymbol{q})\,F_n(\boldsymbol{q}_1,\ldots,\boldsymbol{q}_n)\delta^{(1)}(\boldsymbol{q}_1)\ldots\delta^{(1)}(\boldsymbol{q}_n)$$



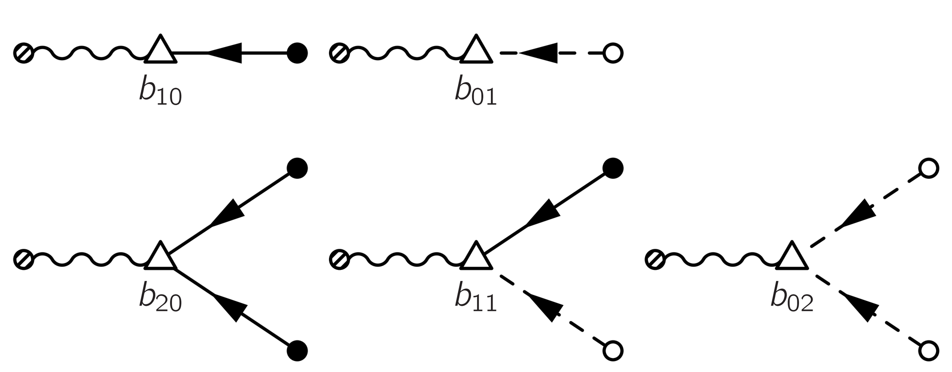
- Multivariate Biasing [Giannantonio & Porciani 2010]
  - In presence of primordial non-Gaussianity, the number denisty of collapsed objects depends not only on the underlying dark matter distribution but also on the amplitude of the primordial gaussian potential.

$$\delta_{\rm g}(\boldsymbol{x}) = \sum_{i,j} b_{ij}\delta^i(\boldsymbol{x})\varphi^j(\boldsymbol{x})$$

  - In k-space this leads to a convolution of matter fields and potentials, which can be interpreted as mode coupling vertices with the constant vertex factor b_{ij}.

$$\delta_{{\rm g},ij}(\boldsymbol{k}) = b_{ij}\int d^3q_1 \ldots \int d^3q_{i+j}\,\delta^{(\rm D)}\left(\boldsymbol{k} - \sum_s \boldsymbol{q}_s\right)\delta_1(\boldsymbol{q}_1)\ldots\varphi_j(\boldsymbol{q}_{i+j})$$

  - In the diagrammtic prescription, matter and potential modes are coupled to give the overdensity of collapsed objects. The simplest vertices are b_{10}, converting a matter field into a tracer field and $b_{01} \propto (b_{10}-1)$, the standard non-Gaussian bias.

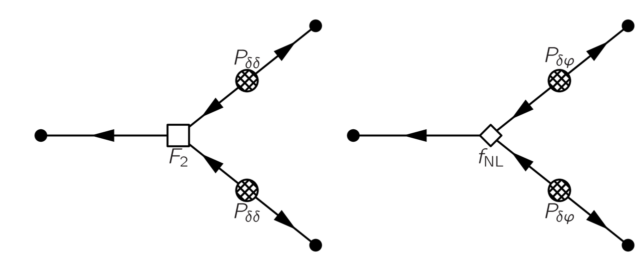


- n-Point statistics
  - Glue n diagrams with i source fields and n outer points in all possible ways and then pair the source fields in all possible ways.
  - Two linked source fields lead to a power spectrum and a momentum conserving delta function

$$\left\langle\delta(\boldsymbol{k})\delta(\boldsymbol{k}')\right\rangle = (2\pi)^3\delta^{(D)}(\boldsymbol{k}+\boldsymbol{k}')P_{\delta\delta}(k)$$

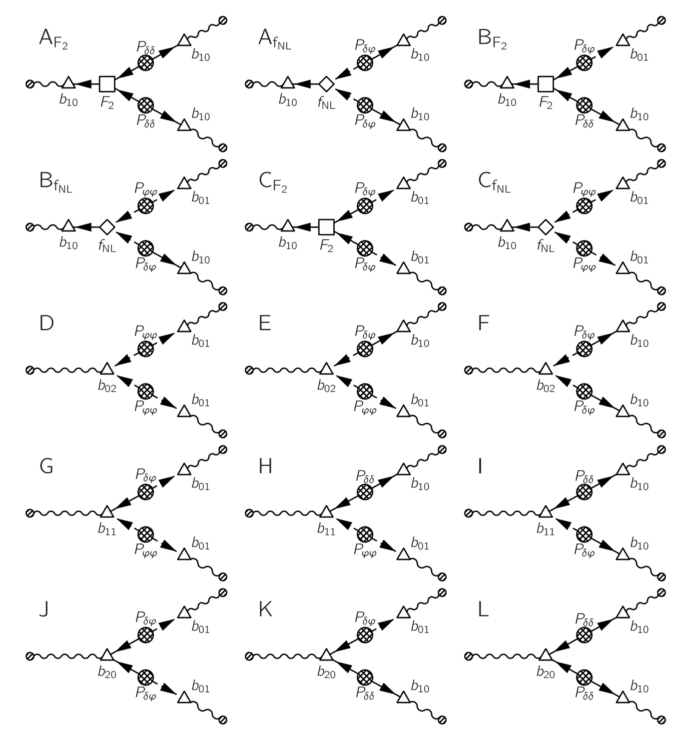
A pedagogic Example: The Matter Bispectrum

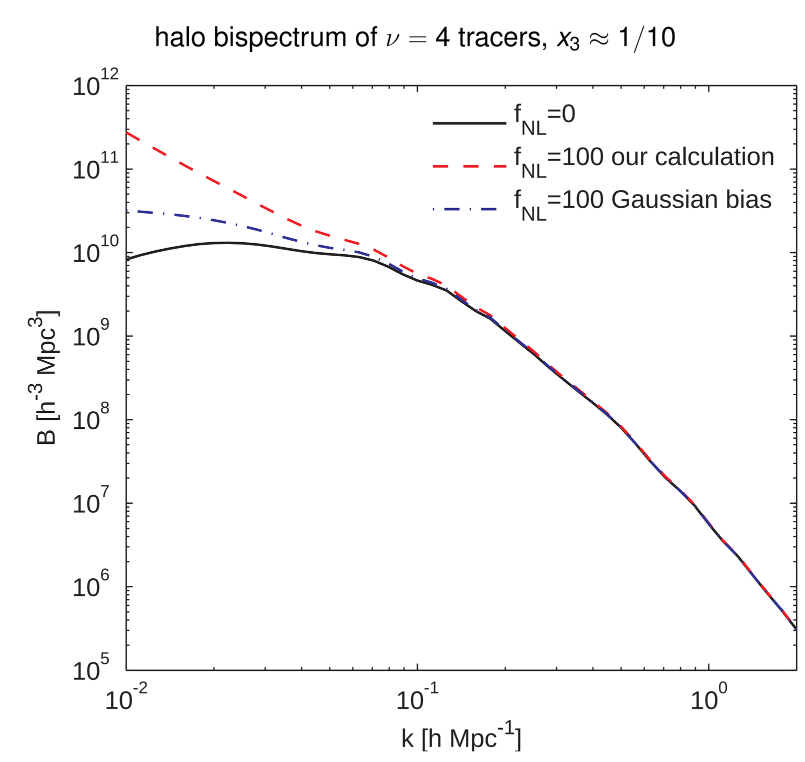
$$\begin{aligned}B_{\rm mmm}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) =& 2P(k_1)P(k_2)F_2(\boldsymbol{k}_1,\boldsymbol{k}_2)\\ &+2f_{\rm NL}\frac{P(k_1)P(k_2)\alpha(k_3)}{\alpha(k_1)\alpha(k_2)} + 2\text{ cyc.}\\ =& B_{F_2}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) + B_{f_{\rm NL}}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)\end{aligned}$$



Galaxy Bispectrum - Squeezed Configuration

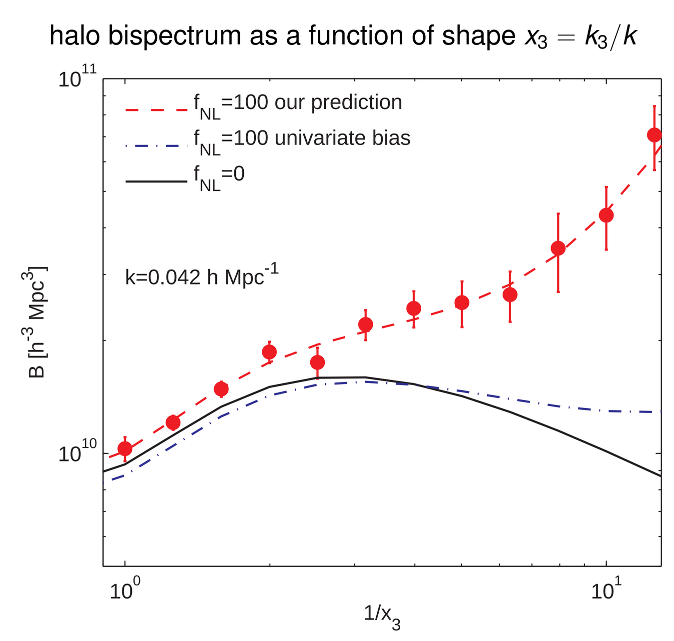
- The general bispectrum depends on an overall scale $k = k_1$ and two angles $x_3 = k_3/k$ and $x_2 = k_2/k$.
- The bispectrum calculated in our approach shows an enhancement with respect to the Gaussian and previously calculated non-Gaussian bispectra in the squeezed limit of an isosceles configuration $k_3 \ll k_1 = k_2$.
- The bias parameters b_{ij} are calculated assuming an universal mass function.

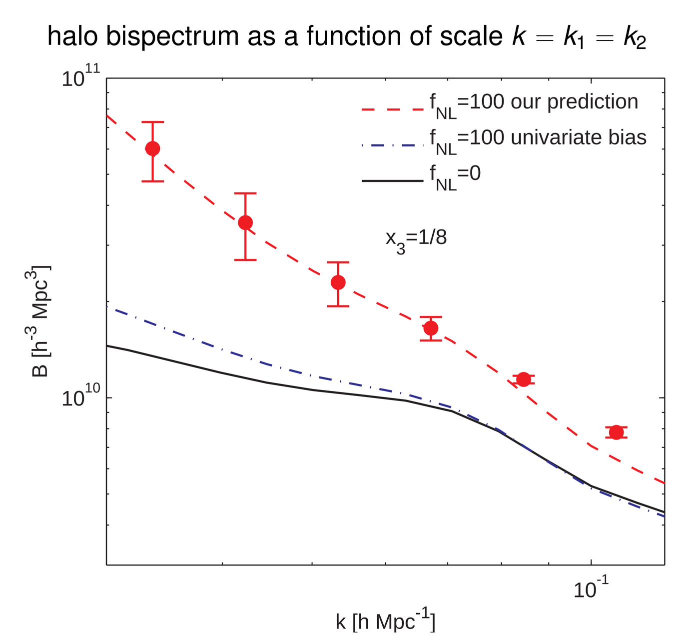




Comparison to Simulations

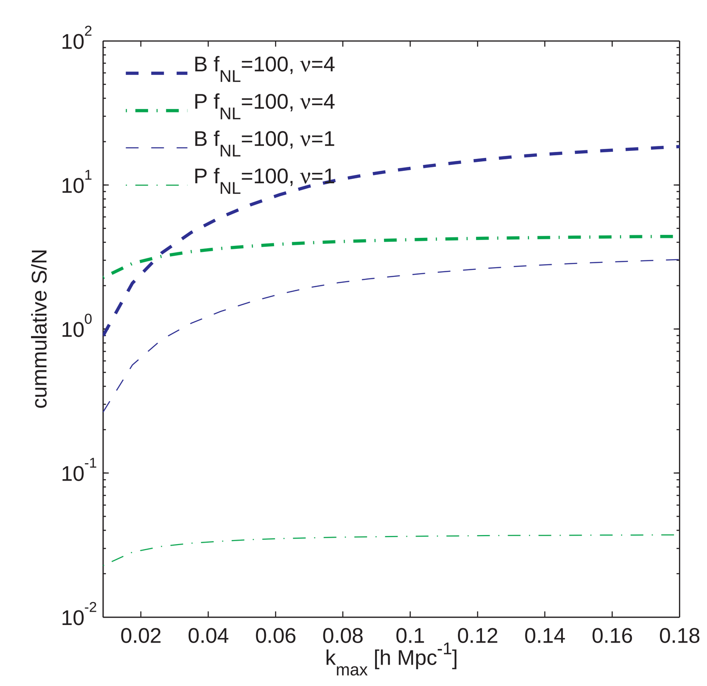
- We compare the results of our model to the simulation measurements of the halo bispectrum by [Nishimichi et al. 2009].
- After adapting the low mass cutoff of the halo sample to reproduce their b_{10}, our model is fully predictable since we calculate the bias factors b_{ij} from the non-Gaussian halo mass function.





Galaxy Bispectrum - Signal-to-Noise

- We calculate the cummulative signal to noise (SN) on $f_{\rm NL}$ for a bispectrum and power spectrum measurement, summing over all triangles up to a certain wavenumber $k_{\rm max}$.
- We consider two halo samples
  - The $\nu = 4$ tracers correspond to $M \approx 1\times10^{14}\ h^{-1}M_\odot$.
  - The $\nu = 1$ tracers correspond to $M \approx 4\times10^{12}\ h^{-1}M_\odot$ haloes and have $b_{10} \approx 1$.
- SN in the bispectrum is exceeding SN of the powerspectrum even for moderately high $k_{\rm max}$



Summary

- Achievements
  - We develop a diagrammatic prescription for the calculation of n-spectra including effects of i) non-linear clustering, ii) biasing and iii) primordial non-Gaussianity.
  - Our model shows corrections to existing Bispectrum calculations at tree level.
  - We show that the constraints on $f_{\rm NL}$ obtainable from the bispectrum analysis exceed the ones obtainable from the power spectrum analysis.
  - Our fully predictive model is in good agreement with numerical simulations.
- Outlook
  - Implementation of loop corrections
  - Extension to other shapes of non-Gaussianity
  - Consideration of relativistic effects