Galaxy Bias in General Relativity

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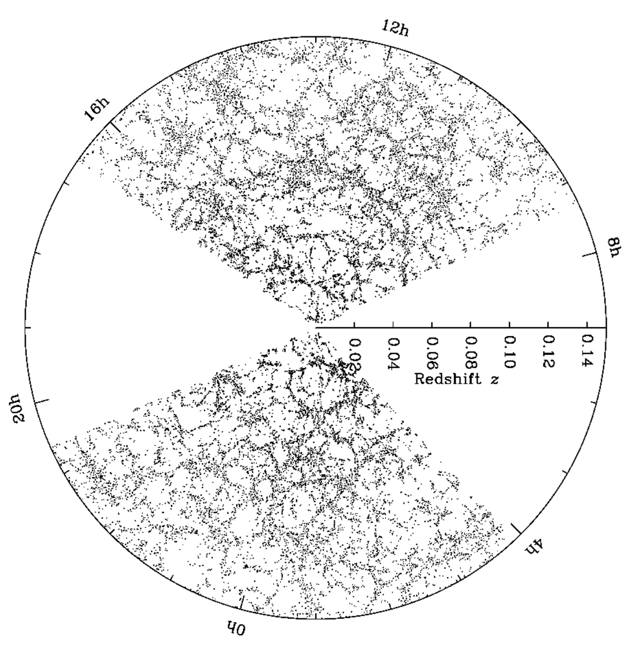
Introduction

- Observables in Cosmology
  - CMB: linear perturbations → Relativistic description
  - LSS: non-linear clustering → Newtonian description
- Current and future dark energy surveys
  - Provide better precission and larger scales
  - Constraints from LSS are complementary to CMB and can serve as cross checks
- Features in the distribution of galaxies
  - Baryon Acoustic Oscillations $k \approx 0.06\ h^{-1}$Mpc
  - Primordial non-Gaussianity $k \approx 0.001\ h^{-1}$Mpc
- Large Scale Simulations increasing in size
  - 6592 h^{-1}Mpc Horizon Run [Kim et al. 2009]



Motivation & Goals

- Motivation
  - Primordial non-Gaussiantity couples long and short modes leading to a remarkable feature in the low k galaxy power spectrum.
  - Simulation boxes and observations are approaching horizon scales $k/H \approx 1$.
- Goals
  - Understand the influence of long modes on the local dynamics in a relativistic framework
  - Find a general relativistic notion of galaxy bias.
  - Develop a formalism to add long wavelength modes to intermediate scale simulations

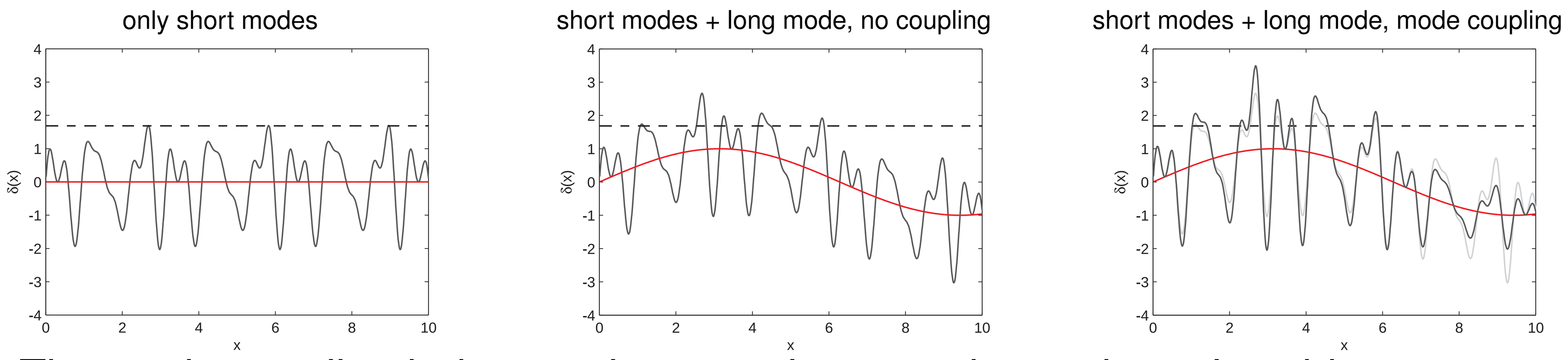
Primordial non-Gaussianity

- Primordial non-Gaussianity is an imprint of the dynamics of the scalar fields driving inflation. Interactions of the fields can lead to a quadratic term in the primordial potential

$$\Phi_{\rm nG} = \Phi_{\rm G} + f_{\rm NL}\left(\Phi_{\rm G}^2 - \left\langle \Phi_{\rm G}^2 \right\rangle\right).$$

- The non-vanishing bispectrum in the squeezed limit leads to mode coupling

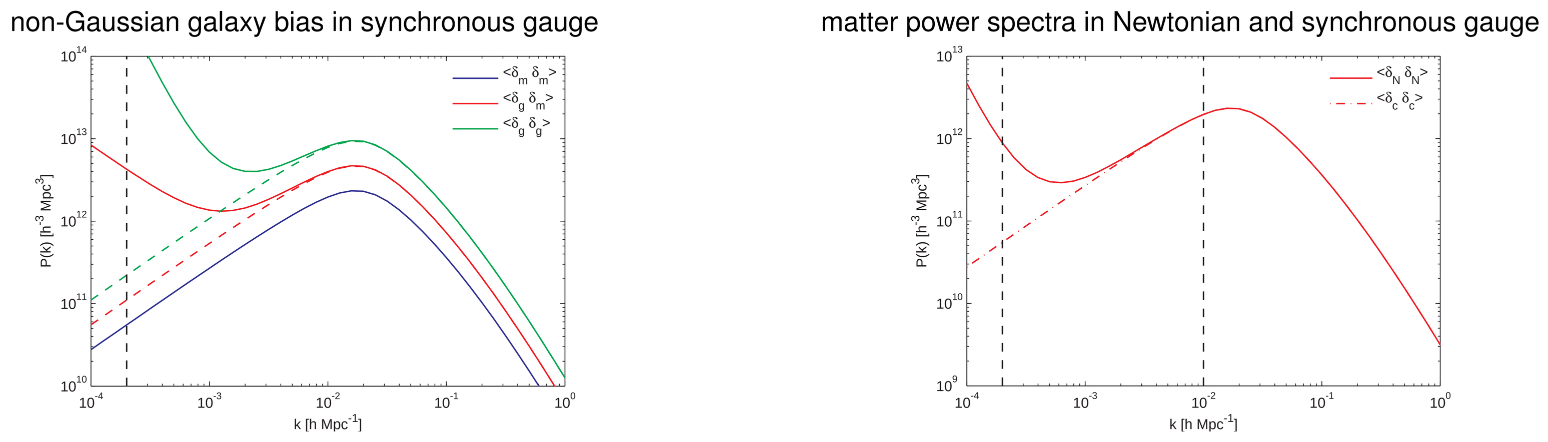
$$\Phi_{\rm s,NG} = (1 + 2 f_{\rm NL}\Phi_{\rm l,G})\Phi_{\rm s,G}.$$



- The mode coupling induces a large scale correction to the galaxy bias

$$\delta_{\rm g} = b_\delta \delta_{\rm m} + b_\Phi \Phi_{\rm l} = \left(b_\delta + 2 f_{\rm NL}\delta_{\rm c}(b_\delta - 1)\frac{H^2}{k^2}\right)\delta_{\rm m}$$

- Studying the non-Gaussian correction dominant on large scales requires some care. ⇒Is there gauge dependence? Is the divergent bias worrisome?



Two Fundamentally Different Regimes

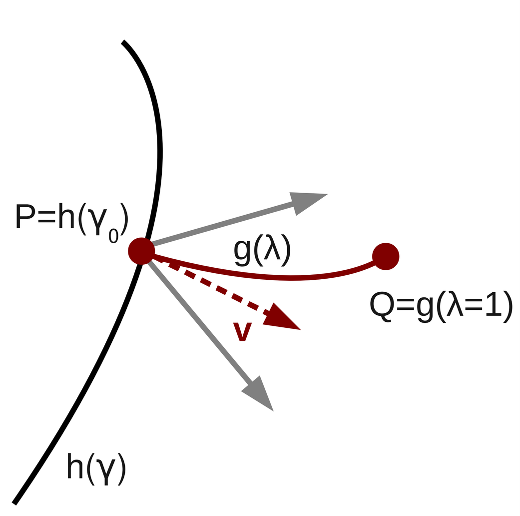
Long modes
- evolve according to linear growth
- relativistic effects for $k/H \approx 1$
- backreaction negligible

Short modes
- non-linear growth
- halo formation → biased tracers
- require numerical simulations

- Usual approach: Studying both regimes requires large scale, high resolution simulations!
- Why not adding long, linear modes by hand and study the response of the short, non-linear modes?

The Local Frame

- Fermi Normal Coordinates [Fermi, 1922]
  - They provide a construction receipe for the local inertial frame and are centered around a time-like geodesic $h(\gamma)$.
  - The metric at $P = h(\gamma_0)$ locally looks rectangular and receives corrections in powers of geodesic distance.
  - The coordinate frame is valid for all time.
- Construction
  - The local observer defines a spatial direction $\boldsymbol{v} = x_{\rm L}^i \boldsymbol{e}_i$,where the Fermi coordinates are the direction cosines $x_{\rm L}^i$, and the Fermi time is the proper time along $h(\gamma)$.
  - Solving for the unique geodesic $g(\lambda)$ tangent to $\boldsymbol{v}$ in P and reaching Q at $\lambda = 1$ provides the mapping between the global and local coordinates of Q.



Long wavelength mode as an effective Curvature

- Consider a global perturbed FRW Universe in Newtonian gauge

$$ds^2 = -\left(1 + 2\Phi\right)dt^2 + a^2(t)\left(1 - 2\Phi\right)d\boldsymbol{x}^2$$

- We identify the effective curvature of the local Fermi frame

$$K = \frac{2}{3}\left[\nabla^2\Phi - \frac{H^2}{\dot H}\left(\nabla^2\Phi + \frac{\nabla^2\dot\Phi}{H}\right)\right] = \frac{2}{3}\nabla^2\zeta = \text{const.}$$

- The expansion of the local frame is described by the effective Hubble rate

$$H_{\rm L} = H + \frac{2}{a^2}\frac{H}{\dot H}\left(\nabla^2\Phi + \frac{\nabla^2\dot\Phi}{H}\right)$$

- The local metric shows quadratic corrections to the Minkowski metric

$$ds^2 = -\left[1 - \left(\dot H_{\rm L} + H_{\rm L}^2\right) r_{\rm L}^2\right]dt_{\rm L}^2 + \left[1 - \frac{1}{2}\left(H_{\rm L}^2 + \frac{K}{a^2}\right) r_{\rm L}^2\right]d\boldsymbol{x}_{\rm L}^2$$

- Long modes can be reinterpreted as curvature in the local frame

Effective Cosmology

- Local density parameters at $a_{\rm L} = 1$

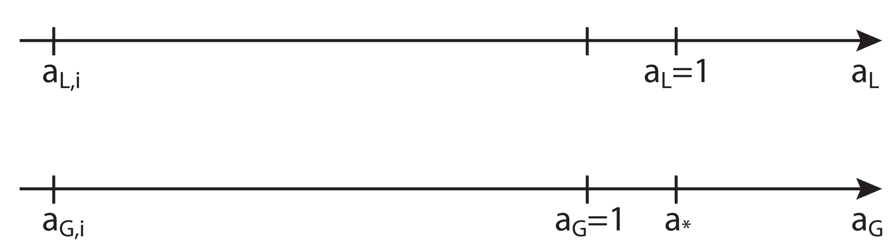
$$\Omega_{\rm m,L} = \frac{8\pi G\bar\rho_{\rm L}}{3H_{\rm L}^2} = \Omega_{\rm m,0}\left[1 + \frac{2}{3}\frac{\nabla^2\zeta}{H_0^2}\right] \qquad \Omega_{\rm K,L} = -\frac{K}{a^2H^2} = -\frac{2}{3}\frac{\nabla^2\zeta}{H_0^2}$$

$$\Omega_{\Lambda,\rm L} = \frac{\Lambda}{3H_{\rm L}^2} = (1 - \Omega_{\rm m,0})\left[1 + \frac{2}{3}\frac{\nabla^2\zeta}{H_0^2}\right] \qquad h_{\rm L} = h\left[1 - \frac{1}{3}\frac{\nabla^2\zeta}{H_0^2}\right]$$

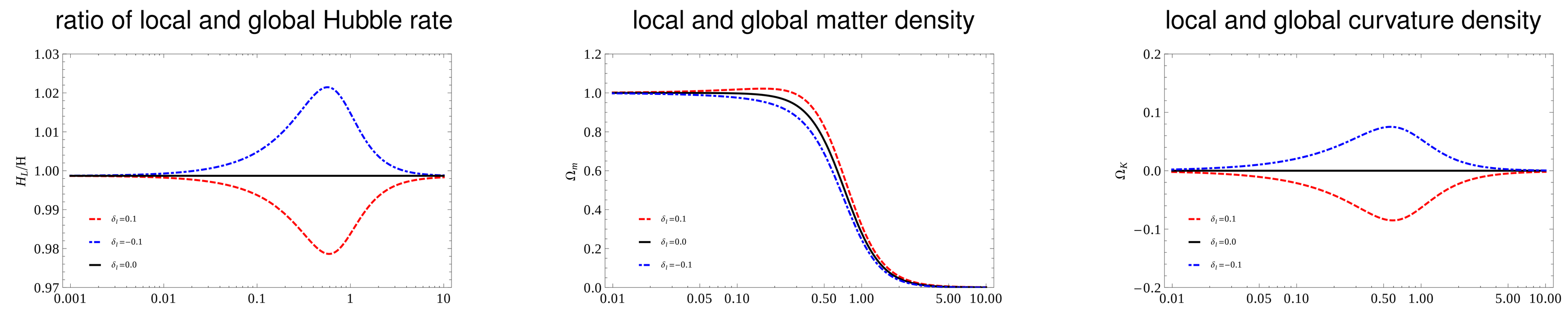
- The local expansion is slowed down by a local overdensity

$$a_{\rm L} = a\left(1 - \frac{1}{3}\delta_{\rm l}^{\rm (S,C)}\right)$$

$$H_{\rm L} = H\left(1 - \frac{f}{3}\delta_{\rm l}^{\rm (S,C)}\right)$$



- Time evolution of the local Universe



Galaxy Bias - Gaussian Case

- Gauge-invariant observed galaxy number density [Yoo et al., 2009]

$$\delta_{n_{\rm obs}}(z,\theta,\phi) = \delta_{n_{\rm p}}^{G}(z,\theta,\phi;\nabla^2\zeta) - \Phi + v^i e_i - (1+z)\frac{\partial}{\partial z}\delta z - 2\frac{1+z}{Hr}\delta z$$
$$- \delta z - 5p\,\delta\mathcal{D}_{\rm L} - 2\kappa + \frac{1+z}{H}\frac{dH}{dz}\delta z + 2\frac{\delta r}{r}$$

- Proper galaxy number density: Bias is the local response to long wavelength perturbations → derivative with respect to the curvature

$$\delta_{n_{\rm p}}(z,\theta,\phi;\nabla^2\zeta) = \frac{1}{n_{\rm p}}\frac{\partial n_{\rm p}}{\partial\nabla^2\zeta}\bigg|_0 \nabla^2\zeta = b_{\nabla^2\zeta}\nabla^2\zeta$$

- Local bias is most naturally described in terms of the comoving-synchronous gauge density perturbation $\delta^{\rm (S,C)} \propto \nabla^2\zeta$

Adding non-Gaussiantity

- Initial Conditions (Long mode $\zeta_{\rm l}$, short mode $\zeta_{\rm s}$)

$$\zeta_{\rm nG} = \underbrace{\zeta_{\rm l,G} + f_{\rm NL}\zeta_{\rm l,G}^2}_{\text{effective Cosmology}} + \zeta_{\rm s,G} + f_{\rm NL}\zeta_{\rm s,G}^2 + \underbrace{2 f_{\rm NL}\zeta_{\rm s,G}\,\zeta_{\rm l,G}}_{\text{long-short Coupling}}$$

- Proper galaxy number density now depends explicitly on primordial curvature perturbation →add a derivative with respect to the long mode amplitude ζ

$$\delta_{n_{\rm p}}(z,\theta,\phi;\zeta) = \frac{1}{n_{\rm p}}\frac{\partial n_{\rm p}}{\partial\nabla^2\zeta}\bigg|_0 \nabla^2\zeta + \frac{1}{n_{\rm p}}\frac{\partial n_{\rm p}}{\partial\zeta}\bigg|_0 \zeta = b_{\nabla^2\zeta}\nabla^2\zeta + b_\zeta\zeta$$

Summary

- Achievements
  - General Relativistic Mapping between the global and local dynamics
  - Long wavelength modes can be reinterpreted as an effective curvature
  - Comoving-synchronous gauge density perturbation most natural for biasing
  - Full non-linear response to a linear long mode calculable using simulations
- Applications
  - Extract mass dependence of bias from simulations