

Influence of Long Wavelength Modes on Local Dynamics and Galaxy Biasing

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Only two things are infinite,
the universe and human stupidity,
and I'm not sure about the former.

Albert Einstein

1 Introduction

2 Fermi Normal Coordinates

3 Dynamics

4 Discussion & Outlook

Motivation

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- Local non-Gaussianity: long modes couple to the short modes
- Simulation boxes are approaching horizon scales - Do we have to worry about GR effects? [Yoo 2009, Wands & Slosar 2009]
- Can the results of “small” scale simulations be mapped to the case with a long wavelength mode? [Tormen & Bertschinger 1996, Cole 1996]

Goal

- Understand the influence of long modes on the local dynamics in a strictly relativistic framework

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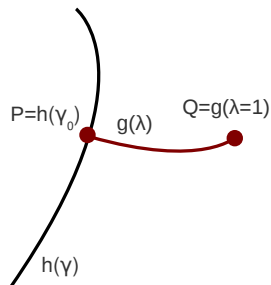
3 Dynamics

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Fermi Normal Coordinates

- WEP - Existence of local inertial frame
- Fermi Normal Coordinates^a - construction of the inertial frame
- centered around a time-like geodesic
- metric locally looks like Minkowski - corrections quadratic in the spatial distance
- Goal: find the mapping from the local to the global dynamics

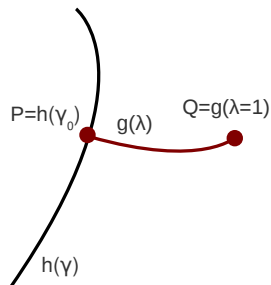
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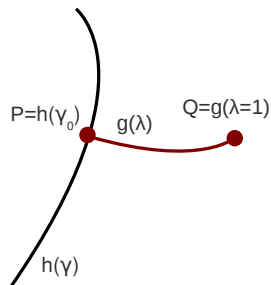
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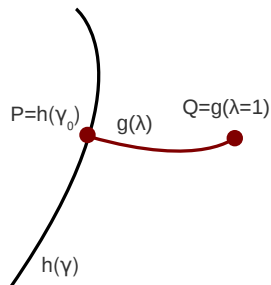
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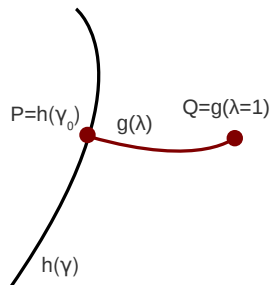
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Fermi Normal Coordinates - Homogeneous FRW

Global Metric

$$ds^2 = -dt^2 + a^2(t) \left(1 + \frac{K}{4} r^2\right)^{-1} dx^2$$

Coordinate Mapping

$$t_{\text{FRW}} = t_{\text{F}} - \frac{H r_{\text{F}}}{2}$$

$$x_{\text{FRW}}^i = \frac{x_{\text{F}}^i}{a(t_{\text{F}})} \left(1 + \frac{H^2 r_{\text{F}}^2}{4}\right)$$

Local Metric

$$ds^2 = -\left(1 - (\dot{H} + H^2) r_{\text{F}}^2\right) dt_{\text{F}}^2 + \left(1 - \frac{1}{2} \left(H^2 + \frac{K}{a^2}\right) r_{\text{F}}^2\right) dx_{\text{F}}^2$$

⇒ Corrections scale as r_{F}/l_H

Fermi Normal Coordinates - Perturbed FRW

Perturbed FRW metric in Comoving-Newtonian Gauge

$$ds^2 = -\left(1 + 2\Phi\right) dt^2 + a^2(t)\left(1 - 2\Phi\right) d\mathbf{x}^2$$

Assume spherical symmetry

$$\Phi(r) = \Phi_0 + \frac{1}{2} \frac{\partial^2 \Phi}{\partial r^2} \Big|_{r=0} r^2 = \Phi_0 + \Phi_2 r^2$$

Local Metric

$$\begin{aligned} ds^2 = & - \left[1 - \left\{ \dot{H} + H^2 - 2(H^2 + \dot{H})\Phi_0 - 3H\dot{\Phi} \right. \right. \\ & \left. \left. - \ddot{\Phi} - 2(H\dot{H} + \ddot{H}) \int \Phi_0 dt' - \frac{\Phi_2}{a^2} \right\} r_F^2 \right] dt^2 \\ & + \left[1 - \left\{ \frac{H^2}{2} - H^2\Phi_0 - H\dot{\Phi} - H\dot{H} \int \Phi_0 dt' + \frac{\Phi_2}{a^2} \right\} r_F^2 \right] d\mathbf{x}^2 \end{aligned}$$

Long wavelength mode as an effective Curvature

Effective Curvature

$$K = 4 \left[\Phi_2 - \frac{H^2}{\dot{H}} \left(\Phi_2 + \frac{\dot{\Phi}_2}{H} \right) \right] = 2\zeta(\mathbf{0}, t_L)_{,rr}$$

Effective Expansion

$$\begin{aligned} \tilde{H}(t_F) = & H(t_F)(1 - \Phi_0(t_F)) - \dot{H}(t_F) \int_0^{t_F} \Phi_0(t') dt' \\ & - \dot{\Phi}_0(t_F) + \frac{2}{a^2(t_F)} \frac{H(t_F)}{\dot{H}(t_F)} \left(\Phi_2 + \frac{\dot{\Phi}_2}{H} \right) \end{aligned}$$

Local Metric

$$ds^2 = - \left[1 - \left(\dot{\tilde{H}} + \tilde{H}^2 \right) r_F^2 \right] dt_F^2 + \left[1 - \frac{1}{2} \left(\tilde{H}^2 + \frac{K}{a^2} \right) r_F^2 \right] d\mathbf{x}_F^2$$

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Dynamics in the Local Frame

Introduce perturbations in the local frame

$$ds^2 = - \left[1 - \left(\dot{\tilde{H}} + \tilde{H}^2 \right) r_{\text{F}}^2 + 2\delta\phi \right] dt_{\text{F}}^2 + \left[1 - \frac{1}{2} \left(\tilde{H}^2 + \frac{K}{a^2} \right) r_{\text{F}}^2 - 2\delta\phi \right] d\mathbf{x}_{\text{F}}^2$$

Einstein and Conservation Equations \Rightarrow Euler Equations

$$\frac{\partial \delta(\mathbf{x}, \tau)}{\partial \tau} + \theta(\mathbf{x}, \tau) = 0$$

$$\frac{\partial \theta(\mathbf{x}, \tau)}{\partial \tau} + \tilde{\mathcal{H}}(\tau) \theta(\mathbf{x}, \tau) + \frac{3}{2} \Omega_{\text{m}} \tilde{\mathcal{H}}^2(\tau) \delta(\mathbf{x}, \tau) = 0$$

Influence of Curvature on the Growth

$$\ddot{\delta} + 2\tilde{H}\dot{\delta} - 4\pi G\tilde{\rho}\delta = 0$$

Rescaling of the Mean Density

$$\tilde{\rho} = (1 + 3\delta_I)\bar{\rho}$$

Influence of Curvature on the linear growth factor

$$\frac{\partial D}{\partial \Omega_K} = \frac{5}{4} \frac{\Omega_m}{a^2} \frac{H_0^2}{\tilde{H}} \int_0^a \frac{da'}{(a'H(a'))^3} - \frac{15}{4} \Omega_m \tilde{H} \tilde{H}_0^2 \int_0^a \frac{da'}{(a'H(a'))^5} \stackrel{EDS}{=} -\frac{4}{7} a^2$$

Rescaling of the Overdensity

$$\tilde{\delta} = \left(1 + \frac{13}{7}\delta_I\right) \delta_0$$

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Achievements

- General Relativistic Mapping between the global and local dynamics
- Long wavelength modes alter the local expansion history and curvature

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