Influence of Long Wavelength Modes on Local Dynamics and Galaxy Biasing

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Only two things are infinite,
the universe and human stupidity,
and I'm not sure about the former.

Albert Einstein

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2 Fermi Normal Coordinates

3 Dynamics

4 Discussion & Outlook

Motivation

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- Local non-Gaussiantity: long modes couple to the short modes
- Simulation boxes are approaching horizon scales - Do we have to worry about GR effects? [Yoo 2009, Wands & Slosar 2009]
- Can the results of "small" scale simulations be mapped to the case with a long wavelength mode? [Tormen & Bertschinger 1996, Cole 1996]

Goal

- Understand the influence of long modes on the local dynamics in a strictly relativistic framework

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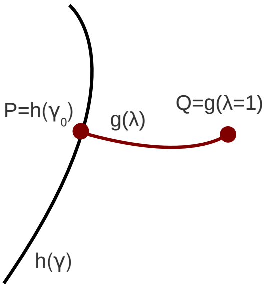
Fermi Normal Coordinates

- WEP - Existence of local inertial frame
- Fermi Normal Coordinates[a] - construction of the inertial frame
- centered around a time-like geodesic
- metric locally looks like Minkowski - corrections quadratic in the spatial distance
- Goal: find the mapping from the local to the global dynamics



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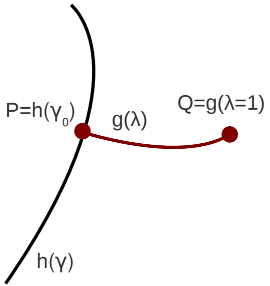

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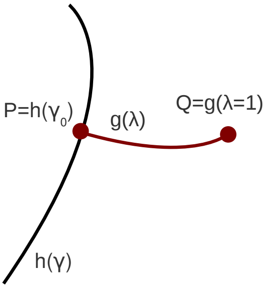

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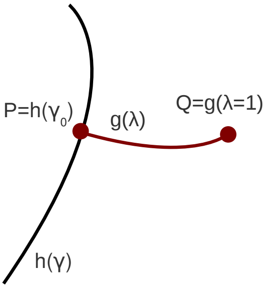

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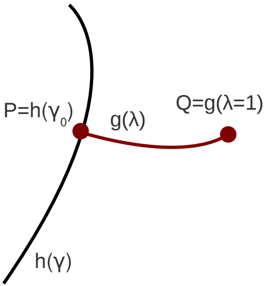

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Fermi Normal Coordinates - Homogeneous FRW

Global Metric

$$ds^2 = -dt^2 + a^2(t)\left(1 + \frac{K}{4} r^2\right)^{-1} d\mathbf{x}^2$$

Coordinate Mapping

$$t_{\mathrm{FRW}} = t_{\mathrm{F}} - \frac{H r_{\mathrm{F}}}{2}$$
$$x^i_{\mathrm{FRW}} = \frac{x^i_{\mathrm{F}}}{a(t_{\mathrm{F}})}\left(1 + \frac{H^2 r_{\mathrm{F}}^2}{4}\right)$$

Local Metric

$$ds^2 = -\left(1 - (\dot{H} + H^2) r_{\mathrm{F}}^2\right) dt_{\mathrm{F}}^2 + \left(1 - \frac{1}{2}\left(H^2 + \frac{K}{a^2}\right) r_{\mathrm{F}}^2\right) d\mathbf{x}_{\mathrm{F}}^2$$

$\Rightarrow$ Corrections scale as r_{F}/l_H

Fermi Normal Coordinates - Perturbed FRW

Perturbed FRW metric in Comoving-Newtonian Gauge

$$ds^2 = -\Big(1 + 2\Phi\Big)dt^2 + a^2(t)\Big(1 - 2\Phi\Big)d\mathbf{x}^2$$

Assume spherical symmetry

$$\Phi(r) = \Phi_0 + \frac{1}{2}\frac{\partial^2\Phi}{\partial r^2}\bigg|_{r=0} r^2 = \Phi_0 + \Phi_2 r^2$$

Local Metric

$$\begin{aligned} ds^2 = &- \Big[1 - \Big\{\dot{H} + H^2 - 2(H^2 + \dot{H})\Phi_0 - 3H\dot{\Phi} \\ &- \ddot{\Phi} - 2(H\dot{H} + \ddot{H})\int \Phi_0 dt' - \frac{\Phi_2}{a^2}\Big\} r_F^2\Big] dt^2 \\ &+ \Big[1 - \Big\{\frac{H^2}{2} - H^2\Phi_0 - H\dot{\Phi} - H\dot{H}\int \Phi_0 dt' + \frac{\Phi_2}{a^2}\Big\} r_F^2\Big] d\mathbf{x}^2 \end{aligned}$$

Long wavelength mode as an effective Curvature

Effective Curvature

$$K = 4\left[\Phi_2 - \frac{H^2}{\dot{H}}\left(\Phi_2 + \frac{\dot{\Phi}_2}{H}\right)\right] = 2\zeta(\mathbf{0}, t_{\mathrm{L}})_{,rr}$$

Effective Expansion

$$\begin{aligned}\tilde{H}(t_{\mathrm{F}}) =& H(t_{\mathrm{F}})\big(1 - \Phi_0(t_{\mathrm{F}})\big) - \dot{H}(t_{\mathrm{F}}) \int_0^{t_{\mathrm{F}}} \Phi_0(t')\, dt' \\ & - \dot{\Phi}_0(t_{\mathrm{F}}) + \frac{2}{a^2(t_{\mathrm{F}})} \frac{H(t_{\mathrm{F}})}{\dot{H}(t_{\mathrm{F}})}\left(\Phi_2 + \frac{\dot{\Phi}_2}{H}\right)\end{aligned}$$

Local Metric

$$ds^2 = -\left[1 - \left(\dot{\tilde{H}} + \tilde{H}^2\right) r_{\mathrm{F}}^2\right] dt_{\mathrm{F}}^2 + \left[1 - \frac{1}{2}\left(\tilde{H}^2 + \frac{K}{a^2}\right) r_{\mathrm{F}}^2\right] d\mathbf{x}_{\mathrm{F}}^2$$

Dynamics in the Local Frame

Introduce perturbations in the local frame

$$ds^2 = -\left[1 - \left(\dot{\tilde{H}} + \tilde{H}^2\right) r_{\mathrm{F}}^2 + 2\delta\phi\right] dt_{\mathrm{F}}^2 + \left[1 - \frac{1}{2}\left(\tilde{H}^2 + \frac{K}{a^2}\right) r_{\mathrm{F}}^2 - 2\delta\phi\right] d\mathbf{x}_{\mathrm{F}}^2$$

Einstein and Conservation Equations $\Rightarrow$ Euler Equations

$$\frac{\partial \delta(\mathbf{x}, \tau)}{\partial \tau} + \theta(\mathbf{x}, \tau) = 0$$

$$\frac{\partial \theta(\mathbf{x}, \tau)}{\partial \tau} + \tilde{\mathcal{H}}(\tau)\theta(\mathbf{x}, \tau) + \frac{3}{2}\Omega_{\mathrm{m}}\tilde{\mathcal{H}}^2(\tau)\delta(\mathbf{x}, \tau) = 0$$

Influence of Curvature on the Growth

$$\ddot{\tilde{\delta}} + 2\tilde{H}\dot{\tilde{\delta}} - 4\pi G\tilde{\rho}\tilde{\delta} = 0$$

Rescaling of the Mean Density

$$\tilde{\rho} = (1 + 3\delta_l)\bar{\rho}$$

Influence of Curvature on the linear growth factor

$$\frac{\partial D}{\partial \Omega_\mathrm{K}} = \frac{5}{4}\frac{\Omega_\mathrm{m}}{a^2}\frac{H_0^2}{\tilde{H}}\int_0^a \frac{da'}{(a'H(a'))^3} - \frac{15}{4}\Omega_\mathrm{m}\tilde{H}\tilde{H}_0^2\int_0^a \frac{da'}{(a'H(a'))^5} \overset{EDS}{=} -\frac{4}{7}a^2$$

Rescaling of the Overdensity

$$\tilde{\delta} = \left(1 + \frac{13}{7}\delta_l\right)\tilde{\delta}_0$$

Discussion

Achievements

- General Relativistic Mapping between the global and local dynamics
- Long wavelength modes alter the local expansion history and curvature

To Do

- Extend to non-Gaussianity
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