Astrophysical Thinking FS 2017

hand in by October 19th

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Question 4: Dynamical Friction (Yoo)

A test particle is moving from an infinity with the relative velocity $v_{\rm rel}$ at an impact parameter b from an object of mass M at rest. Due to the gravitational interaction, the object will induce a small gravitational perturbation $|\delta v_\perp|$ along the perpendicular direction, and its amplitude is

$$|\delta v_\perp| = \frac{2GM}{bv_{\rm rel}} \ . \tag{1}$$

If you have not done it yet, it would be good to derive this formula!

In reality, the test particle will have mass m, and everybody is moving! An object of mass M gravitationally interacts with a smaller object of mass m, inducing a small perturbation in the perpendicular velocity. Since gravity conserves energy, the massive object slightly loses energy. Using the impluse approximation and the energy conservation, find the formula for the slow down:

$$\delta v_\parallel^M = \Delta v_\parallel \frac{m}{M+m} = \frac{2G^2(M+m)m}{b^2 v_{\rm rel}^3} \ . \tag{2}$$

Note that the calculation in Eq. (1) is performed in the center-of-mass frame, but you have to convert it into the lab frame. One may decide to focus on the dimensional alaysis, rather than on deriving the exact formulas.

Consider a distribution of perturbers of mass m, i.e., the number density of such perturbers with the velocity v_m is

$$dn_m = f(v_m) d^3 v_m \ . \tag{3}$$

So the number of perturbers moving with v_m at the impact parameter b at a given time interval Δt is

$$\Delta N = 2\pi b \ db \ v_{\rm rel} \Delta t \ f(v_m) d^3 v_m \ , \tag{4}$$

and by integrating over the impact parameter, we can derive

$$\frac{d}{dt} v_M = -4\pi G^2 M m \ln\Lambda \times f(v_m) d^3 v_m \ \frac{v_{\rm rel}}{v_{\rm rel}^3} \ , \tag{5}$$

where $M \gg m$ is assumed. The fudge factor is called the Coulomb logarithm. Since it is in logarithm, its correct number is less important. Discuss why.

Assuming the isotropic velocity distribution for the perturbers, we can derive the Chandrasekhar dynamical friction formula:

$$\frac{d}{dt}v_M = -16\pi^2 G^2 M m \ln\Lambda \; v_M \times \left[\frac{1}{v_M^3}\int_0^{v_M} dv_m f(v_m) v_m^2\right] \; . \tag{6}$$

Understand the physics behind the formula. What are the asymptotic behaviours of this formula? Consider the case with $v_M \simeq 0$ and the case with $v_M \gg v_m$.

For a Maxwellian velocity distribution,

$$f(v_m) = \frac{n}{(2\pi\sigma^2)^{3/2}} \exp\left[-\frac{1}{2}\frac{v_m^2}{\sigma^2}\right] \; , \tag{7}$$

the formula can be further simplified as

$$\frac{d}{dt}v_M = -\frac{4\pi \ln\Lambda G^2 M m n}{v_M^3} v_M \left[\mathrm{erf}(X) - \frac{2X}{\sqrt{\pi}} e^{-X^2}\right] \; , \qquad X \equiv \frac{v_M}{\sqrt{2}\sigma} \; . \tag{8}$$

In a singular isothermal halo

$$\rho = \frac{v_c^2}{4\pi G r^2} \; , \qquad\qquad v_c = \sqrt{2}\sigma \; , \tag{9}$$

the massive object is also moving at the circular velocity. In such case, the Chandrasekhar formua becomes

$$F \simeq -0.428 \ln\Lambda \frac{GM^2}{r^2} \; , \tag{10}$$

and the force is independent of its host halo mass. Based on this, the time scale for the spiral-in is

$$t_{\rm df} = \frac{1.17}{\ln\Lambda}\frac{r_{\rm ini}^2 v_c}{GM} = \frac{2.64\times10^{11}}{\ln\Lambda}\left(\frac{r_{\rm ini}}{2\ \rm kpc}\right)^2\left(\frac{v_c}{250 \rm km\,s^{-1}}\right)\left(\frac{M}{10^6 M_\odot}\right)^{-1} \rm yr \; . \tag{11}$$

Derive the formulas.

(Application #1) How long would the LMC take to spiral in? You need some numbers.

(Application #2) Consider a merger of two halos, both of which are described by a singular isothermal profile. As the smaller object spirals in, it will lose its mass in the outskirt beyond the tidal radius.[1] What would happen to this system? Will they merge or the satellite lose mass enough to stay afloat?

(Application #3) Suppose our Galaxy is made of many massive black holes instead of dark matter particles. Due to the gravitational interactions with

[1] The tidal force basically deals with the mean enclosed density. Understand this.

these black holes, the Galactic disk will heat up. In the Solar neighborhood, the mass density is $\rho_m = 10^{-2} M_\odot \text{ pc}^{-2}$, and the velocity dispersion of the Galactic disk is $\sigma \simeq 80 \text{ km s}^{-1}$. What is the maximum mass of black holes that is consistent with these numbers?

$$\Delta\left(\sigma^2\right) = \frac{8\pi G^2 M \rho_m \ln\Lambda}{v_M}\Delta t \ . \tag{12}$$

What assumptions are needed to reach this formula?