Astrophysical Thinking HS 2018

hand in by February 22nd

February 19, 2018

Question 1: WIMP Miracle (Yoo)

- no need to look up any text books! no need to type your answers!

The matter content of the Universe is dominated by dark matter. Any successful candidates for dark matter should satisfy, at least, the following conditions: (1) electrically neutral (or dark), (2) stable with lifetime longer or comparable to the age of the Universe, (3) mass density is around $\Omega_m = \rho_{\rm dm}/\rho_c \simeq 0.3$, where ρ_c is the critical density.

Weakly interacting massive particles (WIMP) are leading candidates for dark matter, and WIMP stands for any exotic (not yet known) massive particles that only weak interact with other particles.[1] As they weakly interact with ordinary matter, they are thought to be thermally produced in the early Universe, when the temperature is higher than their mass $T \gg m_{\rm dm}$. This scenario is plausible, as it nicely fits in the standard Big Bang theory, where the ordinary matter such as H, He, Li, and so on is also thermally produced. A stable (but unknown) WIMP particle satisfies the first two conditions for dark matter, and we can check what is required for the third condition.

The evolution of the number density is governed by the Boltzmann equation:

$$\frac{d}{dt}n_{\rm dm} + 3Hn_{\rm dm} = \text{coll.} \simeq n_{\rm dm}\Gamma \ , \qquad\qquad \Gamma \simeq n_{\rm dm}\left\langle\sigma v\right\rangle \ , \tag{1}$$

where $\langle\sigma v\rangle$ is the cross-section times the velocity averaged over the thermal distribution. The left-hand side describes the number density dilution due to the expansion and the right-hand side is the collisional term that can create or annihilate. The collision term naturally depends on the detailed collisional process, but order-of-magnitude it has the dimension of collisional rate times the number density.

Therefore, when the collisional rate is high, the number density is in thermal equilibrium with other constituents, such that its number density distribution will follow the Boltzmann distribution. When the collisional rate Γ drops below H as guessed from the equation, it stops interacting and freezes out. The freeze-out temperature T_f can be estimated as

$$\Gamma \simeq H \propto T^2 \quad \rightarrow \quad n_{\rm dm} \simeq \frac{T^2}{\langle\sigma v\rangle} \ \ \text{at } T = T_f \ , \tag{2}$$

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[1] No interaction at all (except gravity) is a possibility, but we do not know any such particles. Particles with strong interactions cannot be dark matter (why?).

where the Hubble parameter $H \propto T^2$ is dominated by radiation (why?).

Since the number density will dilute as the photons, we can estimate its number density today as

$$n_{\rm dm}^{\rm today} \simeq \left(\frac{n_{\rm dm}}{n_\gamma}\right)_f n_\gamma^{\rm today} \ . \tag{3}$$

Assuming that WIMPs are massive, the equilibrium distribution and the freeze-out temperature are (why?)

$$n_{\rm dm} \propto e^{-m_{\rm dm}/T_f} \ , \qquad\qquad T_f \simeq m_{\rm dm} \ . \tag{4}$$

While the thermally averaged $\langle\sigma v\rangle$ is often a function of temperature $\propto T^n$, the exponential factor is the main factor, deciding the freeze-out temperature.

Therefore, the energy density of dark matter today is then

$$\rho_{\rm dm} = m_{\rm dm} n_{\rm dm} \simeq \frac{n_\gamma^{\rm today}}{\langle\sigma v\rangle} \ , \tag{5}$$

and the density parameter is

$$\Omega_m h^2 = \frac{\rho_m}{\rho_c} h^2 \simeq 0.23 \left(\frac{3\times 10^{-26}{\rm cm}^3 s^{-1}}{\langle\sigma v\rangle}\right) \ , \tag{6}$$

where the critical density is (how many protons per cubic centimeter?)

$$\rho_c = \frac{3H_0^2}{8\pi G} = 2.775\times 10^{11} h^2 M_\odot {\rm Mpc}^{-3} = (\quad)\ m_p\ {\rm cm}^{-3} \ , \tag{7}$$

and the Hubble parameter today is

$$H_0 = 100\ h\ {\rm km/s\ Mpc} \ , \qquad\qquad h \simeq 0.7 \ . \tag{8}$$

1) The density parameter $\Omega_m h^2$ is independent of the mass of the WIMP particles, and 2) its value works out nicely for the typical weak interaction $\langle\sigma v\rangle$. Miracle!

- Discussion (1): WIMP miracle would be a miracle, if we had found a WIMP in a lab. So far, the direct detection experiments put constraints on

$$\sigma \le 1\ {\rm zb} \simeq 10^{-45}{\rm cm}^2 \ , \tag{9}$$

depending on mass, and the future experiments will put even stronger constraints. Given this constraint, any viable candidates are not weakly interacting any more. How can we have particles with $\sigma \le 1$ zb as dark matter?

- Discussion (2): Any upper or lower bounds on $m_{\rm dm}$ exist? Can it be any mass? What if it becomes astrophysical objects like small black holes?

- Discussion (3): Why is there h^2 in the density parameter?