

# Astrophysical Thinking 2018

hand in by May 31st

## 1. Planetary $J_2$

The  $J_2$  of an object has simple physical interpretations. Its order of magnitude can be estimated as either

$$J_2 \sim \frac{R_e - R_p}{R_e} \quad (1)$$

where  $R_e$  is the objects equatorial radius (measured perpendicular to the spin axis) and  $R_p$  is the objects polar radius (measured parallel to its spin axis). Alternatively, we can estimate it as

$$J_2 \sim \frac{\text{Rotational Kinetic Energy}}{\text{Gravitational Potential Energy}}. \quad (2)$$

- (a) Use the second equation to derive an analytic expression for  $J_2$  in terms of the spin frequency of the planet ( $\Omega$ ), the mean density of the planet ( $\bar{\rho}$ ), and any natural constants.
- (b) Estimate  $J_2$  for the Earth and for Saturn without looking up the answers directly. Then compare your answers to the truth by looking up the answers directly.
- (c) When will  $J_2$  of a planetary object be zero?