Astrophysical Thinking (FS 2018)

hand in by March 8th

February 28, 2018

Question 3: Cosmological Magnetic Field (Yoo)

• no need to look up any text books! no need to type your answers!

We will study the magnetic fields in cosmology. The magnetic field is commonly localized, and it is often irrelevant in cosmology and large-scale structure, or is it? One way to probe the magnetic field in our Galaxy, for instance, is to use the Faraday rotation of a background light source. A linearly polarized light can be thought of as a combination of two circular polarizations, rotating in the opposite directions. When a linearly polarized light propagates through a medium with a magnetic field, the polarization interacts with the medium and the magnetic field, such that the phase speeds of two circular polarizations change and as a consequence the polarization direction of the linearly polarized light rotates. This phenomenon is called the Faraday rotation, and naturally the level of rotation depends on the frequency of the light.

Assuming that the source is linearly polarized and all the observed frequencies originated from the same source,[1] we can measure the polarization angle Φ as a function of frequency (in fact, the convention is in terms of wavelength λ):

$$\Phi = \Phi_0 + \mathrm{RM}\ \lambda^2 \ , \qquad [\Phi] = \mathrm{rad} \ , \qquad [\mathrm{RM}] = \mathrm{rad}\ m^{-2} \ , \tag{1}$$

where Φ_0 is the unknown initial polarization direction at the source. The RM stands for the rotation measure in radio astronomy:

$$\mathrm{RM} = \frac{e^3}{2\pi m^2 c^4} \int dr\ n_e B_\parallel \simeq 0.81\ \mathrm{rad}\ m^{-2} \left(\frac{n_e}{\mathrm{cm}^{-3}}\right) \left(\frac{B_\parallel}{\mu\mathrm{G}}\right) \left(\frac{L}{\mathrm{pc}}\right) \ , \tag{2}$$

where n_e is the free electron number density, L is the path length, and $B_\parallel$ is the magnetic field parallel to the light propagation direction (• check the dimension). While typical stars are highly magnetized (~ 1 G), the magnetic fields in galaxies are weak ($\sim 1 - 10\ \mu$G) and confined (often to the spiral arms). The magnetic fields in the intergalactic medium (IGM) are even less understood, but thought to be less than 1 nG.

• ***Question.***— Suppose we have some magnetic field on cosmological scales ($\gg$ 1 Mpc), how can we probe this cosmological magnetic field by using the Faraday rotation? What is the farthest light source we can use for large-scale structure? It is the cosmic microwave background (CMB) radiation. Assuming that the

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[1] What would be the counter-example for the second condition?

magnetic field in the Universe is $B \simeq 1$ nG (i.e., IGM: why?), what would be the rotation angle for CMB due to this magnetic field? From Eq. (2), we need to know the distance to the last scattering surface of CMB (well known) and the free electron number density (not so well known), but istead, use the fact that the optical depth is order unity at the last scattering surface: $1 \simeq \int dr\ n_e\sigma_T$. The RM for the root-mean-square rotation angle works out to be

$$\mathrm{RM} \simeq 280 \text{ rad } m^{-2} \ , \tag{3}$$

for CMB at 30 GHz. Note that the Planck satellite has frequency bands over $30-900$ GHz. How many degrees is the rotation angle at 30 GHz? Can this be measured? or its absence thereof can be constrained by the Planck satellite? Indeed, the upper limit is around 1 nG, depending on the models.

- Discussion (1): CMB is linearly polarized, but again we do not know the initial polarization direction Φ_0. Furthermore, which direction shall we look to estimate the rotation angle? Remember CMB is everywhere on the sky.

- Discussion (2): CMB is a powerful cosmological probe, because it largely involves linear theory (no non-linearity), simple atomic physics (no complicated astrophysical objects), and gravity (everyday general relativity!). This Faraday rotation would, if there is any, work against it. How can we be sure that the measured polarization arose from cosmological, not from astrophysical origin?

- Discussion (3): The Faraday rotation angle is proportional to λ^2 (or ν^{-2}). So there exists a clear advantage to go to larger wavelength bands beyond 1 cm (30 GHz). Why does the Planck satellite stop at 30 GHz? What happens at larger wavelength bands (or lower frequency bands)?