Astrophysical Thinking SS 2018

hand in March 15

Question 4: Estimating the dark matter density

- No need to look up any text books! No need to type your answers!

Understanding the nature and properties of dark matter is perhaps the biggest unsolved challenge in astro- and particle physics. Ongoing effort by many research collaborations aim at detecting dark matter directly via its (infrequent) interactions with nuclei of ordinary matter. The recoil rate in such experiments (e.g., LUX, PandaX-II, CDMSLite and many others) is expected to scale proportional to both the interaction cross section and the density of dark matter in the laboratory. Hence, a precise measurement of the cross section is only possible if the dark matter density $\rho_{\rm dm}$ is well constrained.

Various methods have been proposed to infer $\rho_{\rm dm}$ based on the gravitational effect of dark matter. The approach discussed in this question uses the positions and velocities of stars in the disk of the Milky Way (MW). The starting point is the collisionless Boltzmann equation (CBE) that describes dissipationless objects such as stars or dark matter particles

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \nabla_x f \cdot \mathbf{v} - \nabla_v f \cdot \nabla\Phi = 0.$$

Here, $f(\mathbf{x}, \mathbf{v})$ is the distribution function (the phase space density of the dissipationless objects) and $\Phi(\mathbf{x})$ is the gravitational potential. The potential is related to the total mass density ρ via the Poisson equation

$$\nabla^2\Phi = 4\pi G\rho.$$

In cylindrical coordinates

$$\nabla f = \frac{\partial f}{\partial R}\mathrm{e}_R + \frac{1}{R}\frac{\partial f}{\partial \phi}\mathrm{e}_\phi + \frac{\partial f}{\partial z}\mathrm{e}_z,$$

and

$$\nabla^2 f = \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial f}{\partial R}\right) + \frac{1}{R^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}.$$

Use the equations above to write the CBE and Poisson equation in cylindrical coordinates. Then multiply the CBE by v_z, drop any time derivatives (assume steady state), and integrate over the complete velocity space. You should arrive at one of the Jeans-equations:

$$\frac{1}{R}\frac{\partial(R\nu\sigma_{Rz})}{\partial R} + \frac{\partial}{\partial z}\left(\nu\sigma_z^2\right) + \nu\frac{\partial\Phi}{\partial z} = 0$$

What is ν? What are σ_{Rz} and σ_z^2? Provide their equations.

The Poisson equation in cylindrical coordinates is of the form

$$4\pi G\rho = \frac{\partial^2 \Phi}{\partial z^2} + \mathcal{R}.$$

Derive the functional form of $\mathcal{R}$. How large is it in a galaxy with a flat circular velocity curve?

The first term in the Jeans equation is often called the 'tilt'-term. It is found to be small sufficiently close to the disk of the MW. To simplify the analysis let us therefore drop this term. Let us also drop the $\mathcal{R}$ term in the Poisson equation.

Write down the remaining simplified system of equations. Can you come up with a way to use it to measure the dark matter density in the disk of the Milky Way? Which information would you need to measure $\rho_{\rm DM}$? Will it be easier to measure the DM density or the DM surface density?.

Assume the gravitational potential at 1 kpc above the disk relative to the potential in the disk plane is $\sim 2 \times 10^9$ m^2 s^{-2}. Let the surface density of baryons within $\pm$1kpc be $\Sigma_{\rm b} \sim 55\ M_\odot$ pc^{-2}. **Which components do you think contribute most to $\Sigma_{\rm b}$ and which component dominates the overall error in measuring $\Sigma_{\rm b}$? Use the information above to provide an estimate of the both the surface and spatial density of dark matter in the MW disk.**

A very different approach is to use the gas in the disk of the MW to measure the gravitational potential via the hydrostatic equilibrium equation.

$$\nabla P_{\rm gas} = -\rho_{\rm gas} \nabla \Phi.$$

How could this approach be used to determine the density of dark matter? What could be potential obstacles for this method? Can you think of a few?