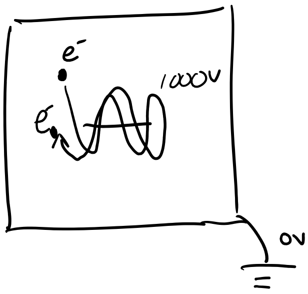
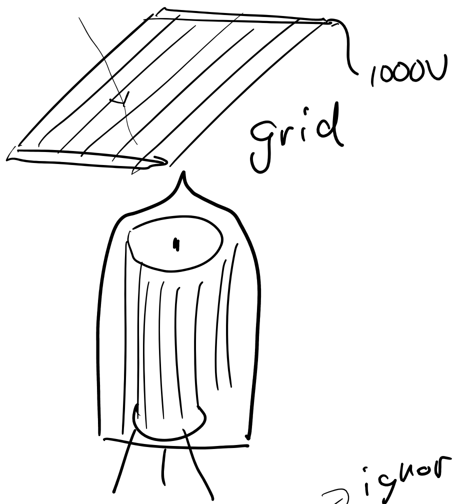
Bi-Linear and Bi-Cubic Interpolation: Electron Beams

Monday, 13 November 2023 11:26





$$\underline{F} = m\,\underline{a} \quad \text{for electron}$$

$-\nabla\phi \rightarrow \underline{F}$ PDE

$\underline{a}$ ← ODE

$$\underline{F} = e(\underline{E} + \cancel{\underline{v_e} \times \underline{B}}) \quad \text{ignore}$$

$$\underline{E} = -\nabla\underline{\Phi}$$

$$\underline{a} = \left(\frac{e}{m_e}\right)(-\nabla\underline{\Phi})$$

$$\ddot{x} = \dot{v}_x = \left(\frac{e}{m_e}\right)\left(-\frac{\partial\Phi}{\partial x}\right)$$

$$\dot{v}_y = \left(\frac{e}{m_e}\right)\left(-\frac{\partial\Phi}{\partial y}\right)$$

$$\dot{x} = v_x$$

$$\dot{y} = v_y$$

System of 4 1^{st}-order ODEs

$$\boxed{\ddot{x} = \frac{d^2x}{dt^2}}$$

Units $e = 1.6 \times 10^{-19}$ C

$m_e = 9.11 \times 10^{-31}$ kg

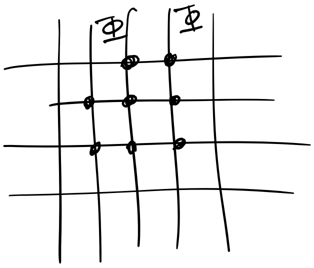
$$\left(\frac{e}{m_e}\right) = 1.76 \times 10^{11}\ C/kg$$

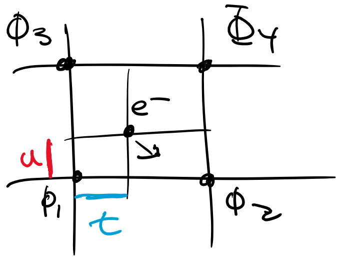
$$\underline{\Phi} = \left[\frac{Nm}{C}\right] \qquad \nabla\underline{\Phi} = \left[\frac{N\cancel{m}}{C\cancel{m}}\right]$$

$$\left(\frac{e}{m_e}\right)(-\nabla\underline{\Phi}) = \left[\frac{C}{kg}\right]\cdot\left[\frac{N}{C}\right] = \frac{N}{kg} = \frac{\cancel{kg}\,m s^{-2}}{\cancel{kg}}$$

$$\ddot{x} = ms^{-2} \checkmark$$

$\ddot{x} = m s^{-2}$ ✓



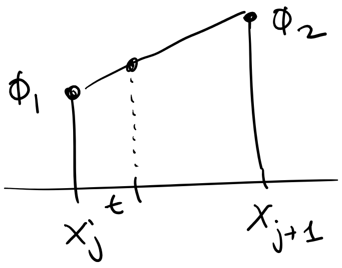


Interpolation

"Mix" these 4 values of the potential to estimate the value at some point inside.

$t \in [0,1]$

$u \in [0,1]$ $\quad t = \dfrac{x - x_j}{\Delta} \quad u = \dfrac{y - y_e}{\Delta}$



$$\phi = t\phi_2 + (1-t)\phi_1$$

if $t=0 \Rightarrow \phi = \phi_1$

$t = 1 \Rightarrow \phi = \phi_2$

$$u = 0 \Rightarrow \phi(x, y_e) = (1-t)\phi_1 + t\phi_2$$

$$u = 1 \Rightarrow \phi(x, y_{e+1}) = (1-t)\phi_3 + t\phi_4$$

$$\phi(x,y) = (1-u)\left[(1-t)\phi_1 + t\phi_2\right] + u\left[(1-t)\phi_3 + t\phi_4\right]$$

$$\boxed{\phi(x,y) = (1-u)(1-t)\phi_1 + (1-u)t\phi_2 + u(1-t)\phi_3 + ut\phi_4}$$

Bi-linear interpolation

$$\left.\frac{\partial \phi}{\partial x}\right|_u = \frac{\partial t}{\partial x} \cdot \frac{\partial \Phi}{\partial t} = \frac{1}{\Delta}\left[-(1-u)\phi_1 + (1-u)\phi_2 - u\phi_3 + u\phi_4\right]$$

$$= \frac{1}{\Delta}\left[(1-u)(\phi_2 - \phi_1) + u(\phi_4 - \phi_3)\right]$$

leave as exercise $\left.\dfrac{\partial \phi}{\partial y}\right|_t$

$$\dot{x} = v_x$$

$$\dot{v}_x = \left(\frac{e}{m_e}\right)\left[-\frac{1}{\Delta}\left((1-u)(\phi_2 - \phi_1) + u(\phi_4 - \phi_3)\right)\right]$$

$$v_x = \left(\frac{-}{m_e}\right)\left[-\frac{1}{a}\left((1-u)(\varphi_2-\varphi_1)+u(\varphi_4-\varphi_3)\right)\right]$$

$$\dot{y} = v_y$$

$$\dot{v}_y = \left(\frac{e}{m_e}\right)\left[\;- - - -\;\right]$$

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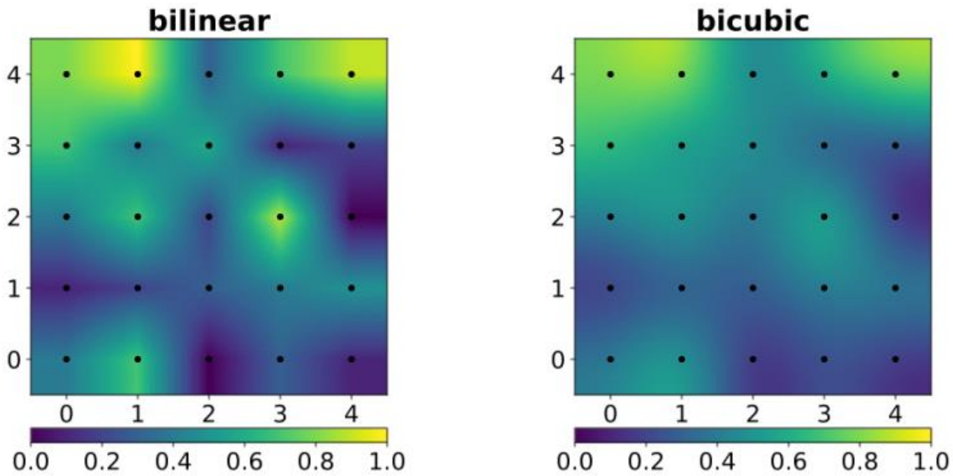
where is the electron (j,e)

RK4 or LeapFrog.

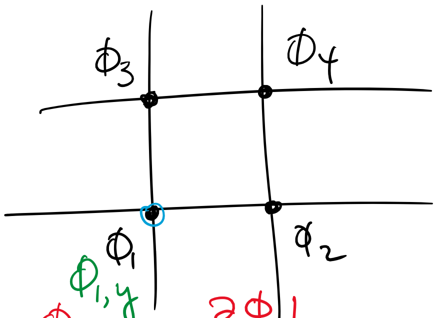


Bicubic:

$$\boxed{\Phi(t,u) = \sum_{i=0}^{3}\sum_{j=0}^{3} a_{ij}\, t^i u^j}$$

$$\begin{bmatrix} a_{00} & a_{01} & \cdots & \\ a_{10} & a_{11} & & \\ \vdots & & a_{22} & \\ & & & a_{33} \end{bmatrix} = M^T \cdot \left[\begin{array}{cc|cc} \Phi_1 & \Phi_2 & \Phi_{1,y} & \Phi_{2,y} \\ \Phi_3 & \Phi_4 & \Phi_{3,y} & \Phi_{4,y} \\ \hline \Phi_{1,x} & \Phi_{2,x} & \Phi_{1,xy} & \Phi_{2,xy} \\ \Phi_{3,x} & \Phi_{4,x} & \Phi_{3,xy} & \Phi_{4,xy} \end{array}\right] \cdot M$$

$$M = \begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & 3 & -2 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$



$\Phi_{1,y}$

$\Phi_{1,x} \equiv \frac{\partial \Phi}{\partial x}\Big|_1$

$\Phi_{1,xy} \equiv \frac{\partial \Phi}{\partial x \partial y}\Big|_{1 \left\{ \begin{smallmatrix} t=0 \\ u=0 \end{smallmatrix} \right\}}$

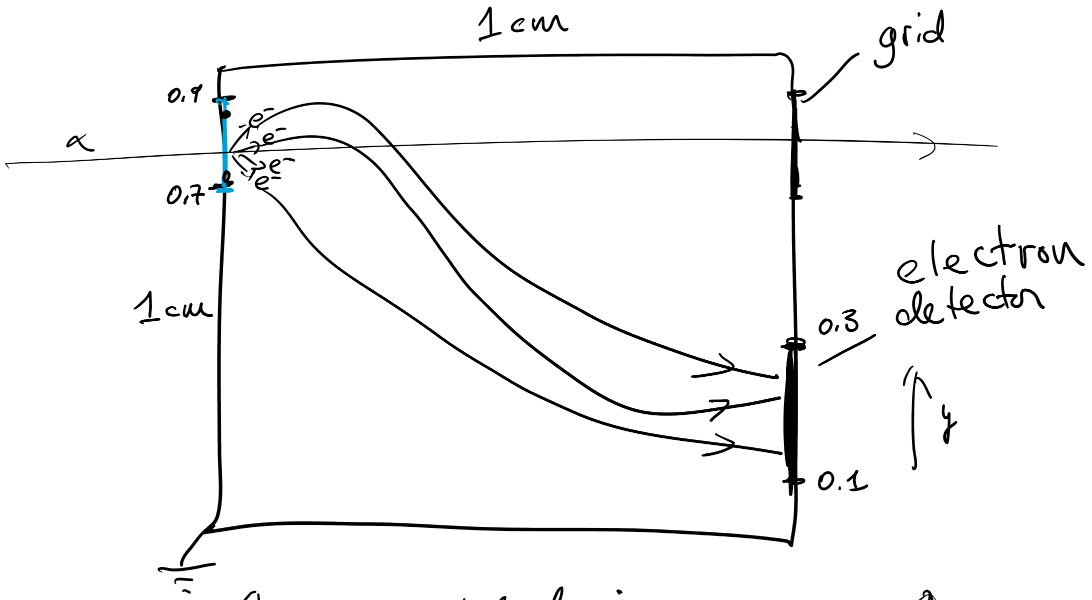
$$\Phi(t,u) = \begin{bmatrix} 1 & t & t^2 & t^3 \end{bmatrix} \cdot \begin{bmatrix} & & \\ & a_{ij} & \\ & & \end{bmatrix} \cdot \begin{bmatrix} 1 \\ u \\ u^2 \\ u^3 \end{bmatrix}$$

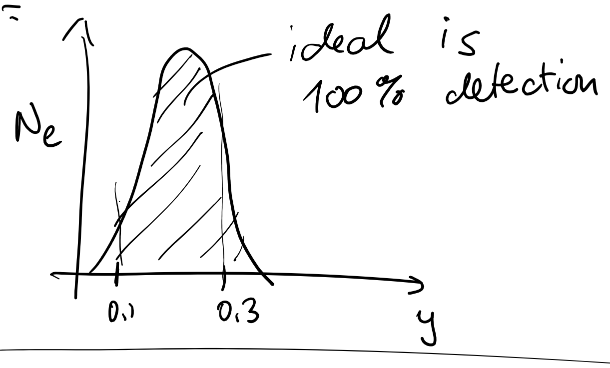
OPTIONAL

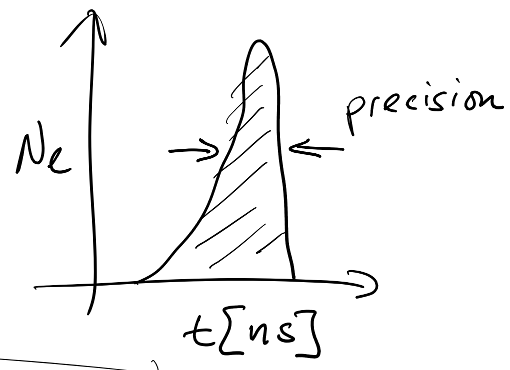
2 Weeks to Prize!!

Week 1 : Code Validation

Week 2 : Designing







t[ns]

$$\boxed{|\underline{v}_e| = 10^6 \text{ m/s} \quad \text{at random angles}}$$

"Draw" boundary conditions

→ Φ, R arrays