Population Growth, Chaos and Fractals

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$$\frac{P_{n+1} - P_n}{P_n} \equiv r$$

or $$P_{n+1} = (1+r) P_n$$

There is a solution for all n and for all values of r,

$$P_n = (1+r)^n P_0$$

Population explosion

Normalize the population with respect to the largest size N that the resources can support,

$$\boxed{p = P/N}$$

p	r
1	0
small	large and positive
~ 1	small
>1	negative!

$$r \propto (1-p) \Longleftarrow$$

$$r = k(1-p)$$

Verhulst Model

$$\frac{p_{n+1} - p_n}{p_n} = k(1-p_n)$$

$$p_{n+1} = p_n + k \boxed{p_n} (1 - \boxed{p_n})$$

Deterministic

$p_n^2 \rightarrow$ Non-linear

Eg, $k = 3$ $\quad p_0 = 0.01$

$p_0' = 0.00999999999\ldots$

$$P_0' = 0.009999999\ldots$$

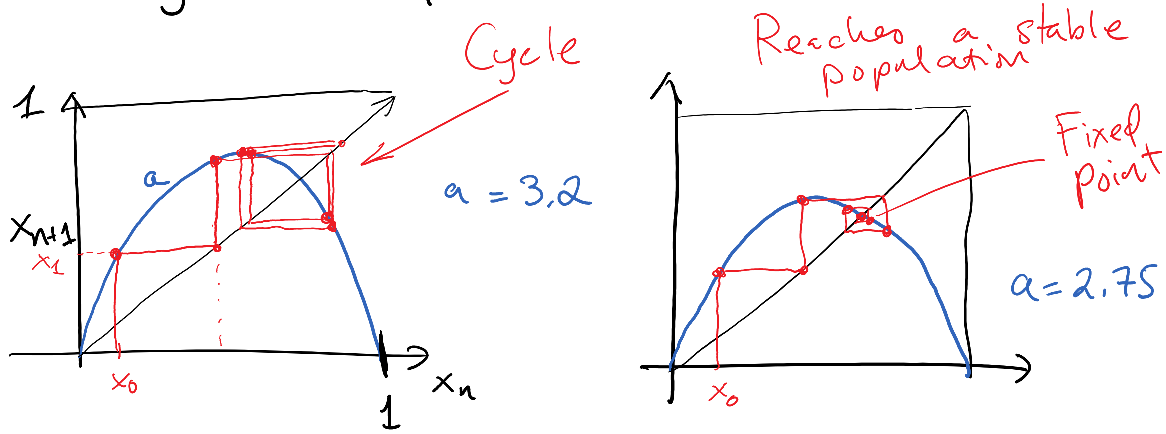
Rewrite in a more convenient form:

$$x_{n+1} = a\,x_n(1-x_n)$$

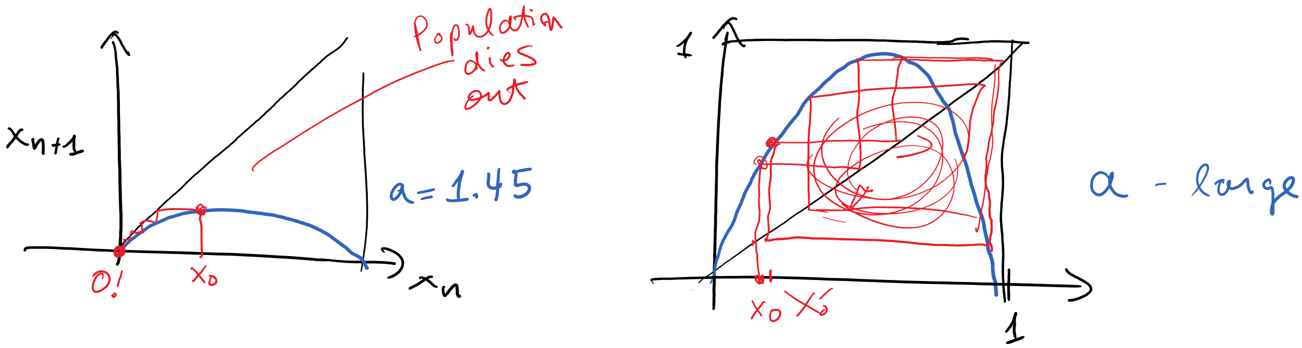
$$x \in [0,1]$$

$$a \in [0,4]$$

Logistic Equation



Independent of the initial x_0!



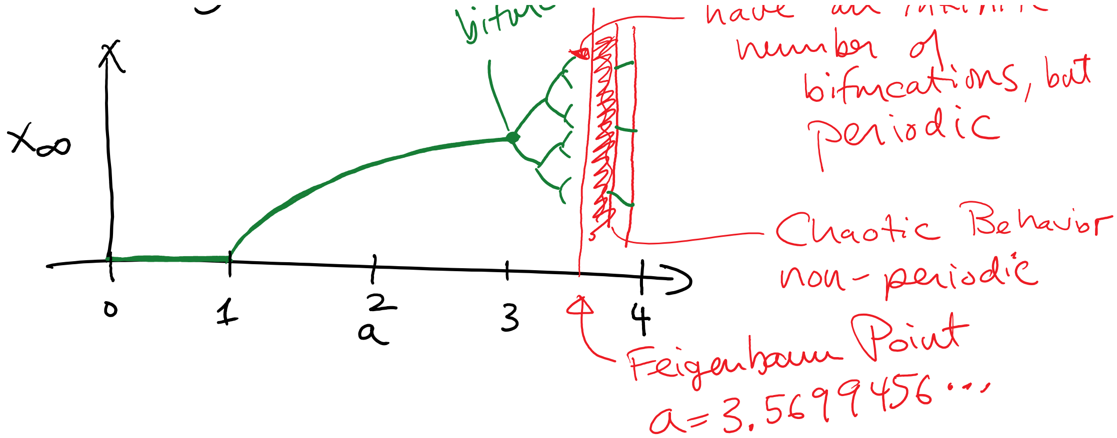
Depends very sensitively on the initial x_0!

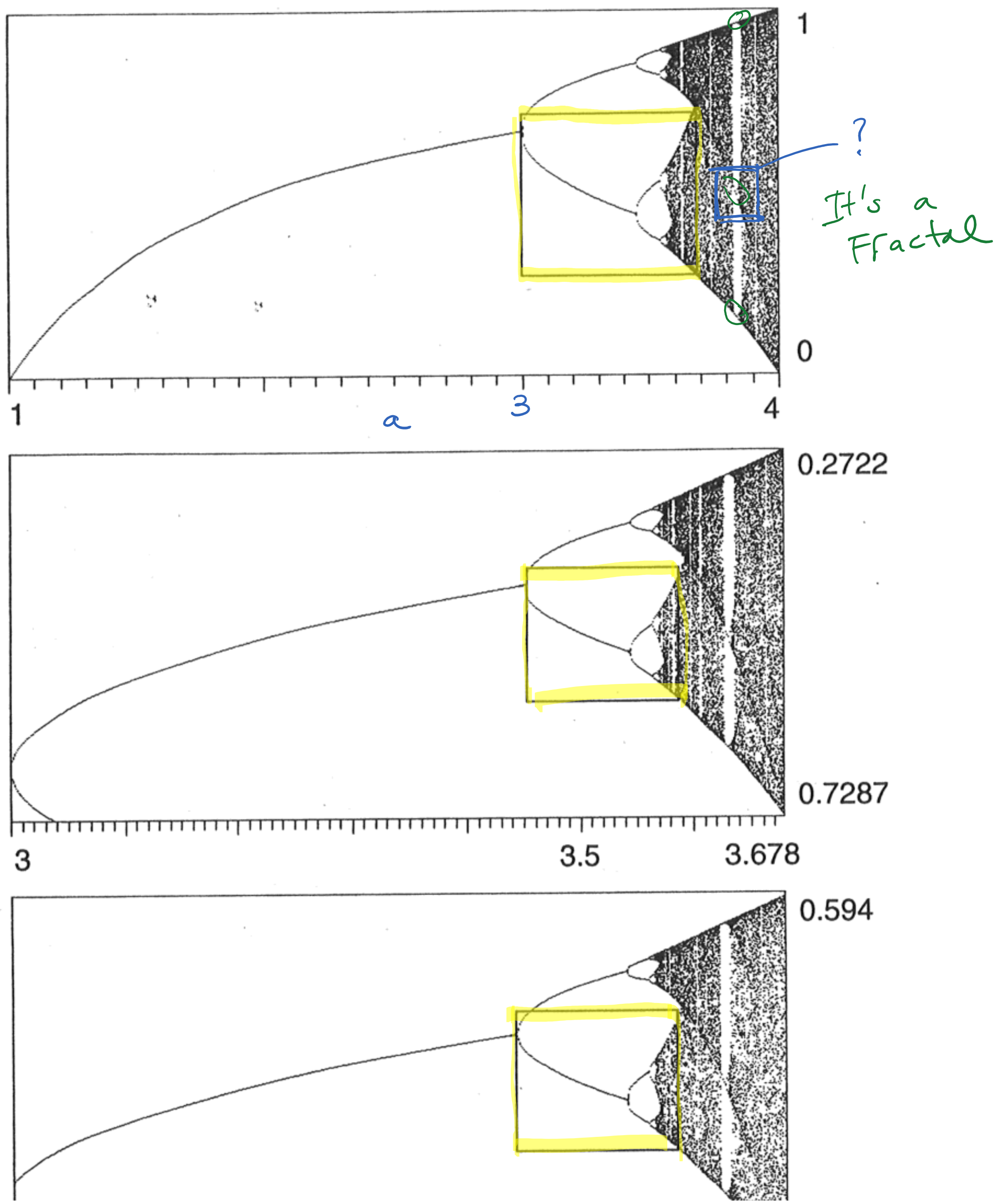
$\boxed{\text{Deterministic Chaos}}$

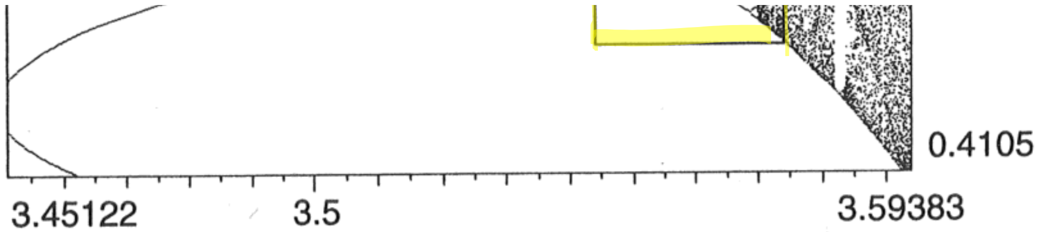
requires a non-linear system.

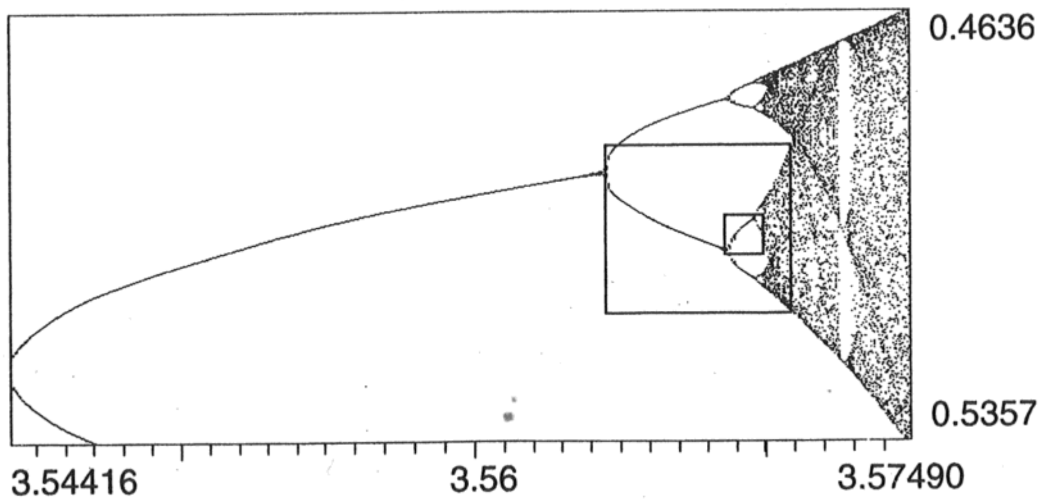
Feigenbaum Diagram

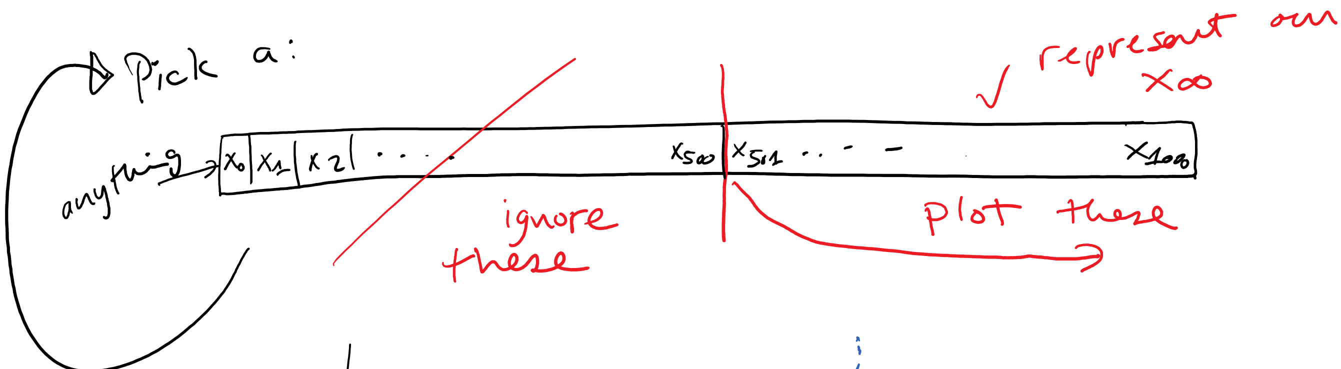
have an infinite number of bifurcations, but

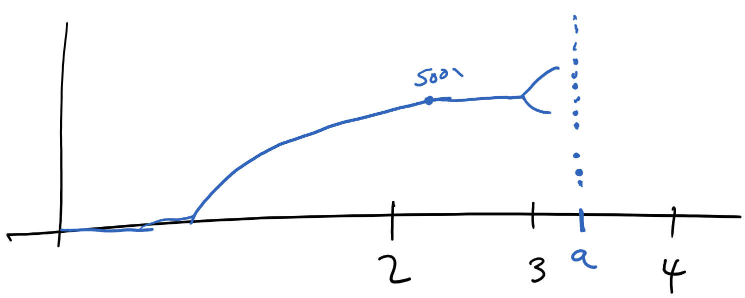

x
x_∞
0
1
2
3
4
a
number of
bifurcations, but
periodic
Chaotic Behavior
non-periodic
Feigenbaum Point
a=3.5699456...


1
0
1
3
4
a
?
It's a Fractal
0.2722
0.7287
3
3.5
3.678
0.594









Next time → iterator Complex number

⇒ Very Beautiful Fractals.