Fractals in Complex Numbers, Beginning of ODEs

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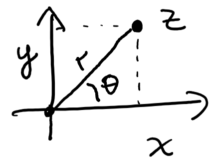
non-linearity: $x_n(1-x_n)$

Complex Numbers: $i^2 = -1$

$$z = (x + iy)$$

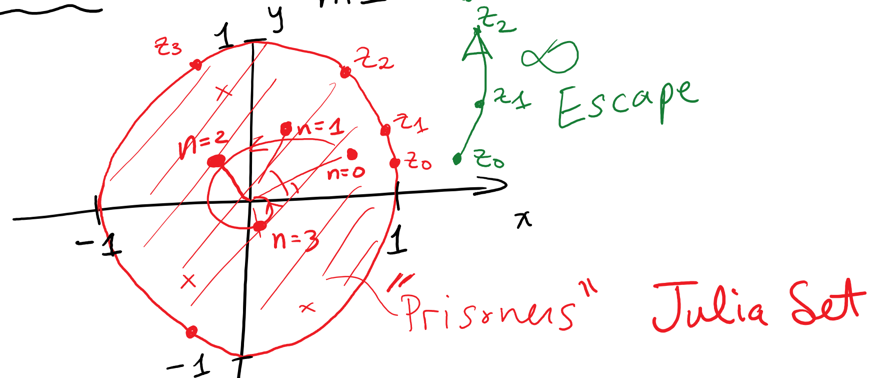
Real Imaginary



$$z = re^{i\theta}$$

let
$$z = x + iy$$
$$w = u + iv$$
$$z \cdot w = (x+iy)(u+iv)$$
$$= (xu - yv) + i(yu + xv)$$
$$z^2 = (re^{i\theta})^2 = r^2 e^{i(2\theta)}$$

Complex Iterator: $z_{n+1} = z_n^2$



A bit boring...

What about $z_{n+1} = z_n^2 + c$

add a complex constant

What is the Julia Set of this Iterator?

For $c = -0.5 + 0.5i$
A connected J_c.

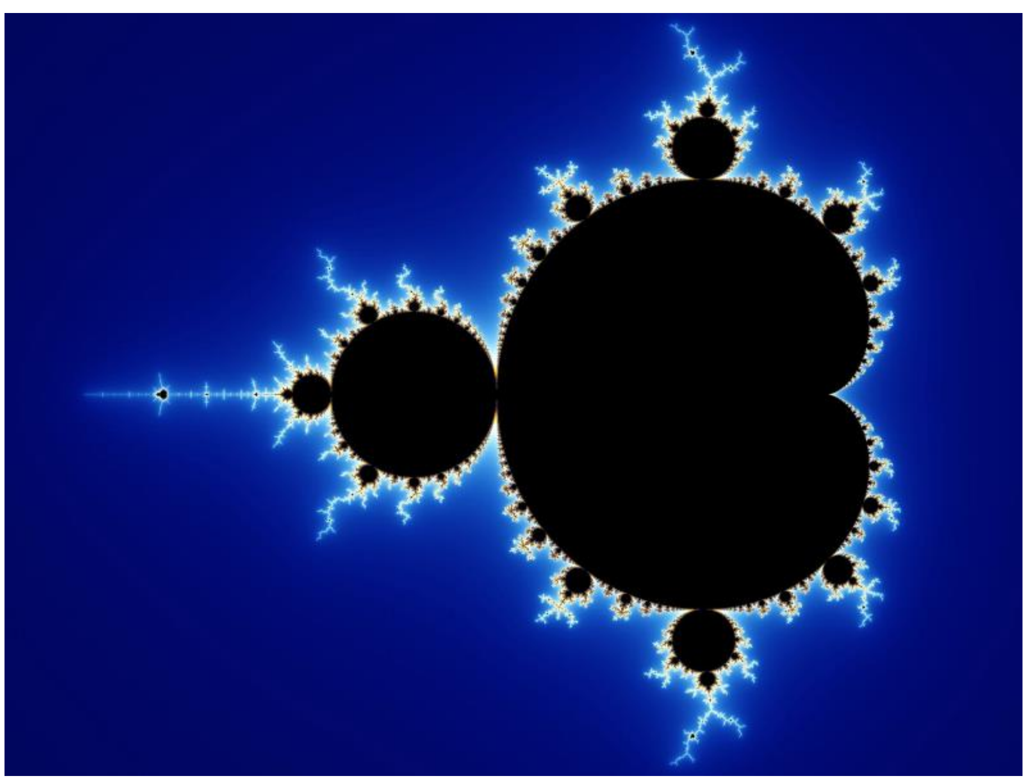
Unconnected J_c
"Dust"

Mandelbrot Set : $M = \{ c \in \mathbb{C} \mid J_c \text{ is connected} \}$

$\equiv$

Another def:

$M = \{ z_0 = c \in \mathbb{C} \mid z_{n+1} = z_n^2 + c < \infty \}$



If $|z_n| > r(c)$ a critical radius, then it will always escape to ∞!

$$r(c) \equiv \max(|c|, 2)$$

Colors: according to how many steps to "escape" that are needed. How "close" to the Mandelbrot set you come.

Do for all pixels in c

```
k = 0
z = c
while (k < 100) {
    if (|z| > r(c)) {
        Draw point with color(k);
        return (c ∉ M);
    }
    z = z*z + c
    k = k+1
}
Draw point with color("Black")
return (c ∈ M);
```

In population growth: $\dfrac{P_{n+1} - P_n}{P_n} = r \qquad \dfrac{1}{P}\dfrac{dP}{dt} = r$

Differential Equations:

ODE: ordinary diff. equations ←

PDE: partial diff. equations

$$\frac{d^2y}{dx^2} + q(x) \cdot \frac{dy}{dx} = r(x)$$

Is an ODE but it is not in simplest form, it's 2^{nd} order.

$$\frac{dy}{dx} = z(x)$$

$$\frac{dz}{dx} \boxed{+ q(x) \cdot z(x)} = r(x) - q(x)z(x)$$

$$\frac{dt}{dx} + q(x)\cdot t(x) = \dots$$

2 × 1st order equations $\frac{d}{dx}$

Generalize this:

$$\frac{dy_i(x)}{dx} = f_i\left(x, y_0(x), y_1(x), \dots, y_{N-1}(x)\right)$$

$i = 0..N-1$

$$\boxed{\frac{d\underline{y}(x)}{dx} = \underline{f}(x, \underline{y})}$$

$$\frac{dy}{dx} = f(x,y)$$

$y(x_{start})$