Population Growth, Chaos & Fractals

Monday, 3 October 2022 12:52

Constant growth rate for a population

$$\frac{P_{n+1} - P_n}{P_n} = r \qquad P_n(P_0) = ?$$

or $P_{n+1} = (1+r)P_n$

There is a solution for all n and r

$$P_{n+1} = (1+r)^n P_0$$ "Population explosion"

Normalize the population :

$$\boxed{p = P/N}$$

"maximal" size

p	r
1	0
small	large & positive
~ 1	small
>1	negative

$r \propto (1-p)$

$r = k(1-p)$

Verhulst Model

$$\frac{p_{n+1} - p_n}{p_n} = k(1-p_n)$$

$$\boxed{p_{n+1} = p_n + k\,\boxed{p_n}(1-\boxed{p_n})}$$

$\propto p_n^2$ Non-linear

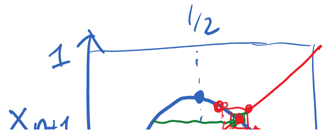
Eg, $k=3$ $\quad p_0 = 0.01$

$p_0' = 0.00999999999\ldots$

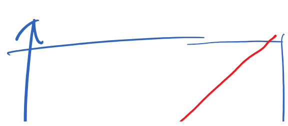
$$X_{n+1} = a X_n (1 - X_n)$$ Logistic Equation

$x \in [0,1]$

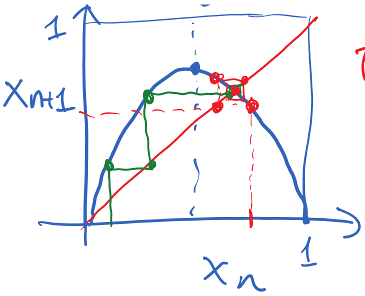
$a \in [0,4]$



Fixed point

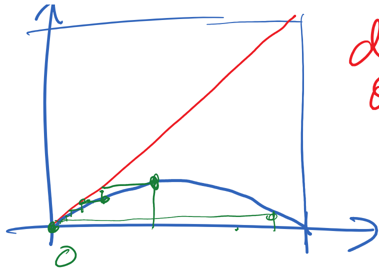


dies out



Fixed point behaviour

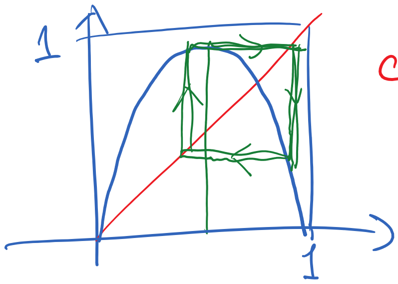
$a = 2.75$



dies out

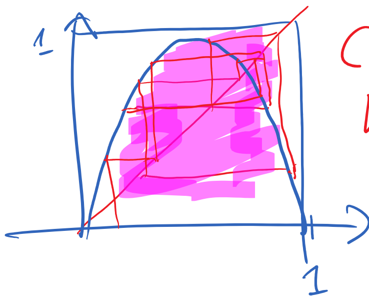
$a = 0.5$

$x_\infty \to 0$



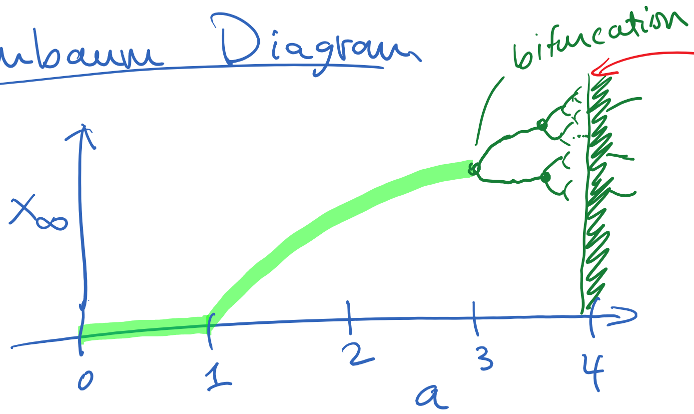
Cyclical behaviour

$a = 3.2$

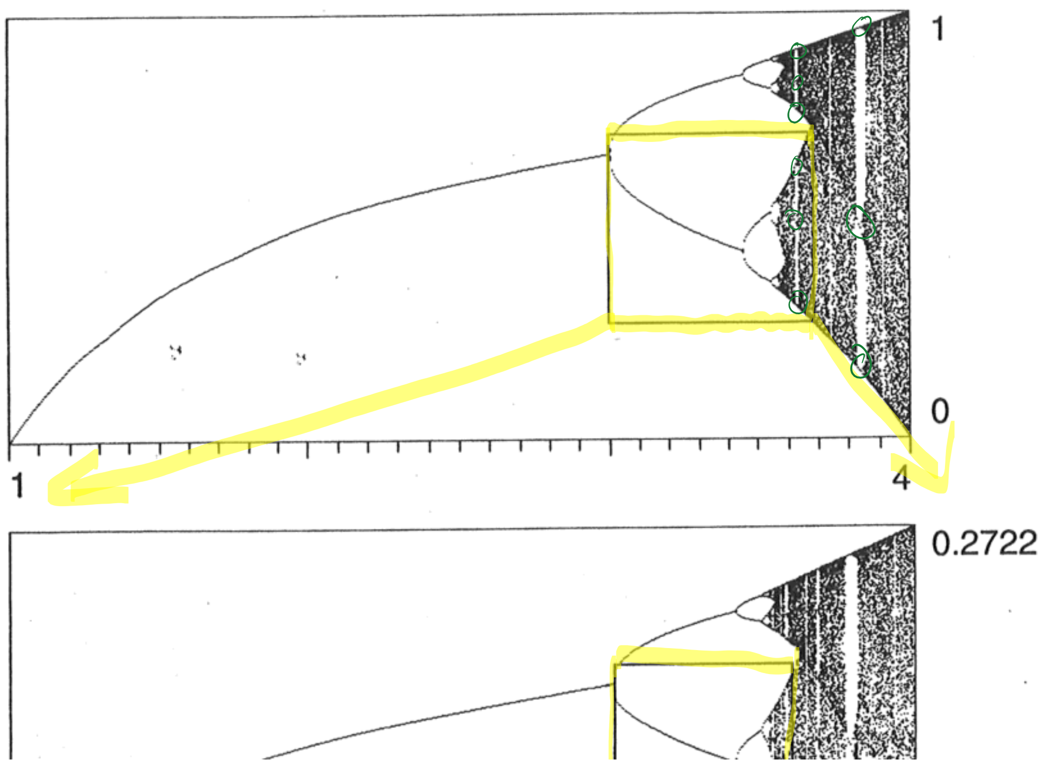


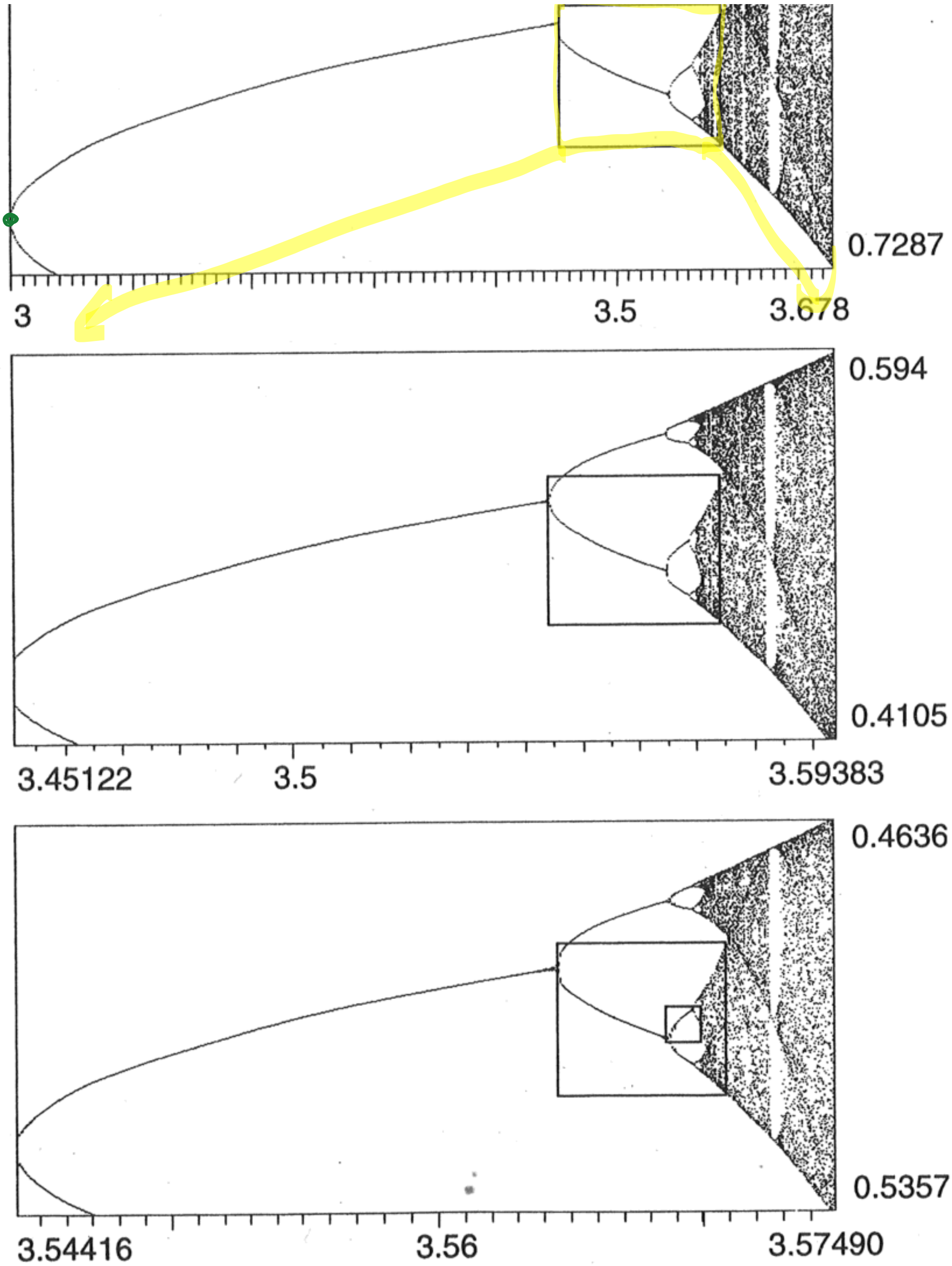
Chaotic behaviour

Feigenbaum Diagram



Feigenbaum point





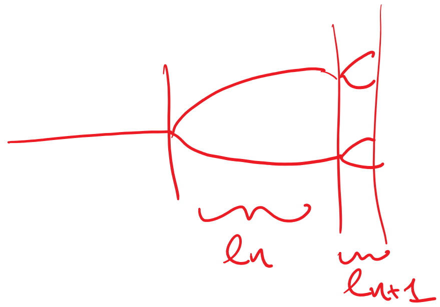
FRACTAL

x_0	x_1	x_2	$\cdots$	x_{500}	x_{501}	$\cdots$	x_{999}

throw them away

Plot <u>all</u> of these for the given a!

UNIVERSAL:



$$\frac{l_n}{l_{n+1}} = 4.6692\ldots$$

Universal property