Fractals using Complex Numbers

Monday, 10 October 2022 10:21

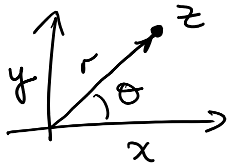
Complex behaviour Non-linearity $x_n(1-x_n)$

Complex Iterators:

$i^2 = -1$

$z = (x + iy)$

Real Imaginary



$z = re^{i\theta}$

let $z = x + iy$

$w = u + iv$

Addition: $z + w = (x+u) + i(y+v)$

Multiplication:

$z \cdot w = (x+iy)(u+iv)$

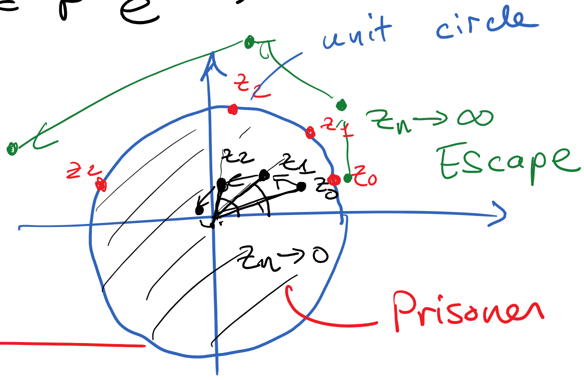
$x \cdot u + ixv + iyu + \underbrace{i^2 yv}_{-yv}$

$= (x \cdot u - yv) + i(xv + yu)$

$z^2 = (re^{i\theta})^2 = r^2 e^{i(2\theta)}$

$z_{n+1} = z_n^2$

Julia Set: the set of prisoners



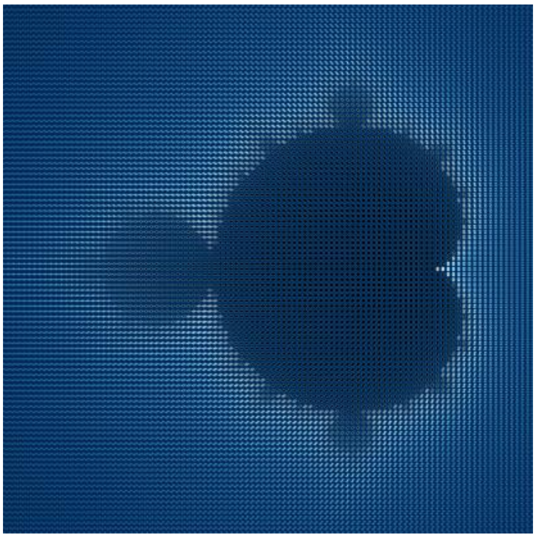
$z_{n+1} = z_n^2 + C$

add a complex constant

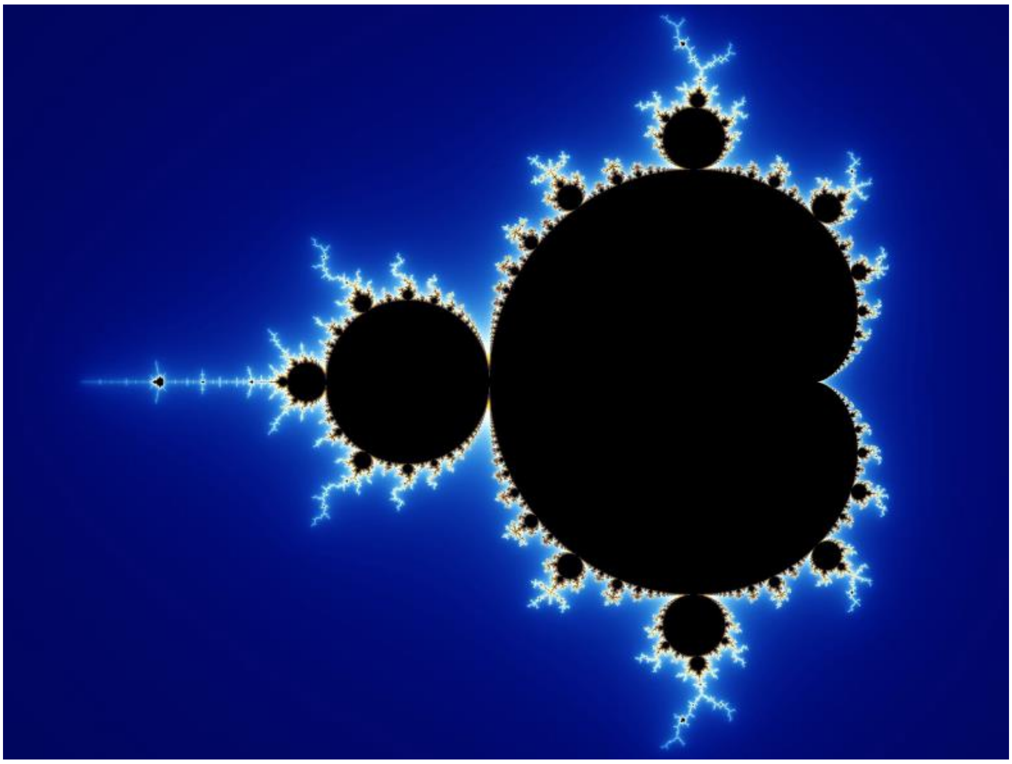
What is the Julia set in this case?

For $C = -0.5 + 0.5i$
A connected set: J_c

$c = ?$
An unconnected set: J_c



A grid of Julia sets, different values of c



Mandelbrot set : Black $\Rightarrow J_c$ is connected
Colored $\Rightarrow J_c$ is unconnected

$$M = \{ c \in \mathbb{C} \mid J_c \text{ is connected} \}$$

Another definition:

$$M = \{ z_0 = c \in \mathbb{C} \mid z_{n+1} = z_n^2 + c < \infty \}$$

if $|z_n| > r(c)$ a critical radius, then it will always escape to infinity!

$$r(c) := \max(|c|, 2)$$

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colors : according to how many steps to escape it takes. How "close" to the Mandelbrot set is an escaping point.

Do for all pixels on the screen (values of c):

```
k=0
z=c
while (k < 100)
      if (|z| > r(c)) then
            Draw point with color(k);
            return(c ∉ M)
      z = z*z + c
      k = k+1
Draw point with color ("black")
return (c ∈ M)
```

Ordinary differential equations (ODEs)

What are differential equations:

Ordinary differential ...

What are differential equations:

$$\boxed{\frac{d^2y}{dx^2} + q(x)\frac{dy}{dx} = r(x)}\ ;\quad 2^{nd} \text{ order}$$

Goal is to find the function $y(x)$

ODE: ordinary D.E. ⇐

PDE: partial D.E. (later in the course)

$$F = ma = m\frac{d^2x}{dt^2} \qquad \text{ODE } 2^{nd} \text{ order}$$

$$\Rightarrow x(t)$$

rewrite

$$\boxed{\begin{aligned} \frac{dy}{dx} &= z(x) \\ \frac{dz}{dx} + q(x)\cdot z(x) &= r(x) \end{aligned}}$$

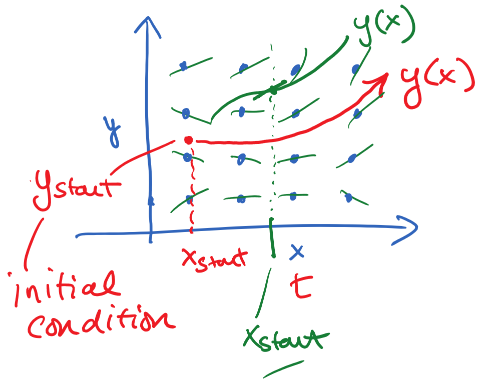
2 times 1^{st} order D.E.

In general we can write:

$$\frac{dy_i}{dx} = f_i(x, y_0(x), y_1(x), \dots, y_{N-1}(x))$$

$$i = 0 .. N-1$$

$$\boxed{\frac{d\underline{y}(x)}{dx} = \underline{f}(x, \underline{y})}$$



$$\frac{dy}{dx} = f(x,y)$$

$$y_i(x_{start}) = y_i^{start}$$

given ↗   ↖ given

Dirchlet Boundary Condition

$$\frac{dy_i(x_{start})}{dx} = \text{Slope given somewhere}$$

(von Neumann Boundary Condition)

Analytical solutions are difficult.

Numerical solutions (algorithm) are <u>easy</u>!

resolution ⇝ errors

track/control errors.