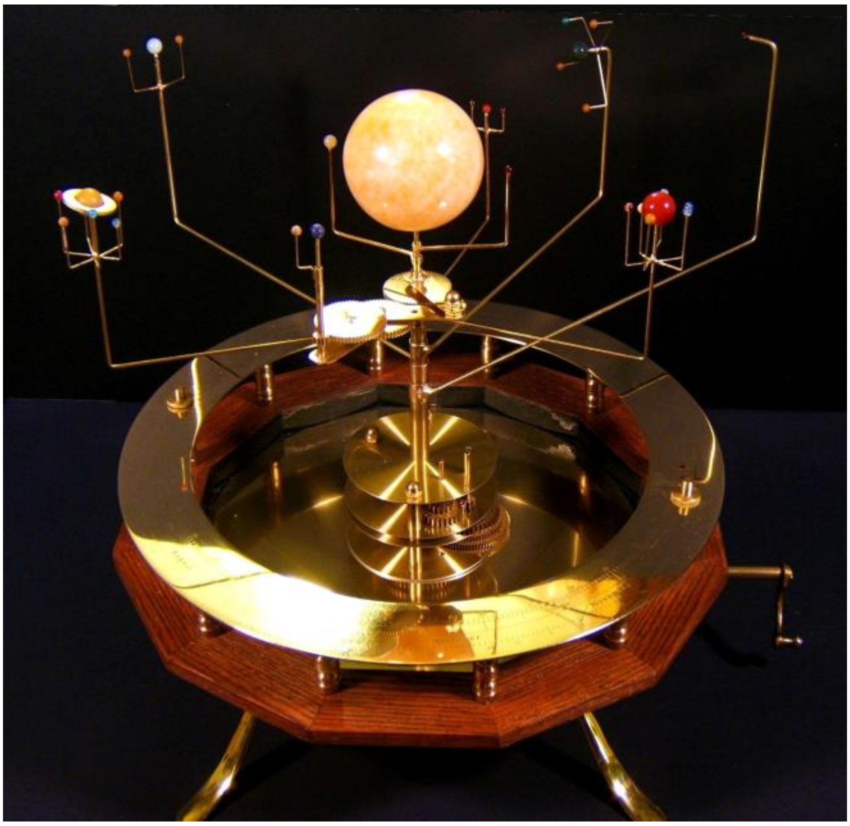
N-body Problem, Simulating the Solar System

Monday, 31 October 2022 12:34



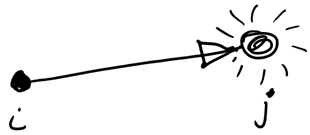
8 planets + Sun $\rightarrow$ 9 bodies

Hamiltonian $H = T + U$

Kinetic Energy

potential Energy

$$\underline{F}_{ij} = \frac{G m_i m_j}{|\underline{r}_j - \underline{r}_i|^2} \cdot \underbrace{\frac{(\underline{r}_j - \underline{r}_i)}{|\underline{r}_j - \underline{r}_i|}}_{\text{unit vector}}$$



$i \neq j$

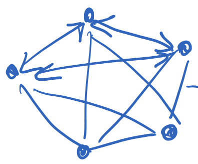
Newton's Law of Gravity

Newton's 3rd Law:

$$\boxed{\underline{F}_{ij} = - \underline{F}_{ji}}$$

How many Forces do we need to calculate?

N-bodies



for each edge here we either have 2 Forces or one interaction

$F_{ij} = -F_{ji}$

5 bodies

$5 \cdot 4 = 20$ Forces

N-bodies $\frac{N(N-1)}{2}$ Interactions to calculate

$O(N^2)$

Imagine how many operations are needed to calculate 10^{12} bodies,

Possible?

Giga $10^9/s$
Tera $10^{12}/s$
Peta $10^{15}/s$
Exaflop $= 10^{18}/s$

$\frac{10^{24}}{\cancel{2}} \cdot \underbrace{\cancel{2}0}_{\approx \text{ number of flops per interaction}} \to 10^{25}$ floating point operations

$\frac{10^{25}}{10^{18}} = 10^7 s = 1/3$ year

$O(N \log N)$ or even $O(N)$

<u>Units</u> $G_N = 6.6742 \times 10^{-11} [m^3 kg^{-1} s^{-1}]$

$G_N \cdot M_\odot = k^2$ Gauss' grav const

$k = 0.01720209895 \, [AU^{3/2} M_\odot^{-1/2} D^{-1}]$

1D = 86400 S.I. seconds

$$\boxed{\underline{F}_i = \sum_{j \neq i} \frac{k^2 m_i m_j}{|\underline{r}_j - \underline{r}_i|^3} (\underline{r}_j - \underline{r}_i)}$$

one of the bodies is the sun $i=0$ say

$$\dot{\underline{p}} = -\frac{\partial H}{\partial \underline{q}} = -\frac{\partial \phi(\underline{q})}{\partial \underline{q}} \equiv -\nabla_{\underline{q}} \phi$$

is just the acceleration

$$\underline{a}_i = \frac{\underline{F}_i}{m_i}$$

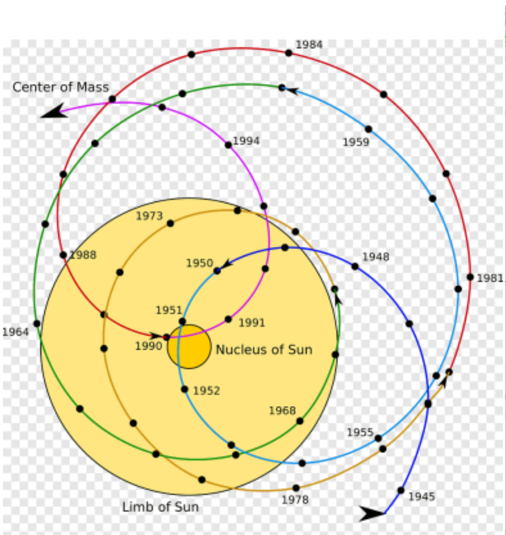
$$\underline{a} = -\nabla \phi$$

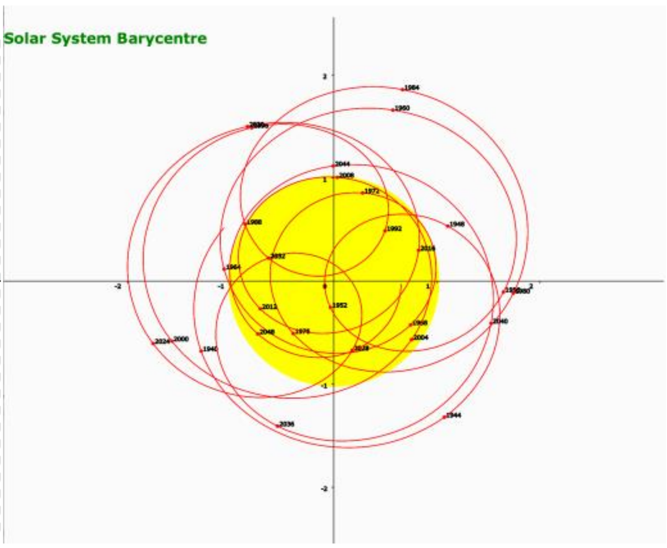
Leap Frog :

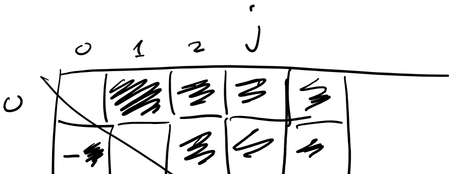
Drift "H = T" (N) $\underline{r}_{i,n+\frac{1}{2}} = \underline{r}_{i,n} + \frac{h}{2} \underline{v}_{i,n}$ $\mathcal{O}(N)$

Kick "H = ϕ" (N) $\underline{v}_{i,n+1} = \underline{v}_{i,n} + \underline{a}_i(\{\underline{r}_{\frac{1}{2}}\})$ $\mathcal{O}(N^2)$

Drift (N) $\underline{r}_{i,n+1} = \underline{r}_{i,n+\frac{1}{2}} + \frac{h}{2} \underline{v}_{i,n+1}$



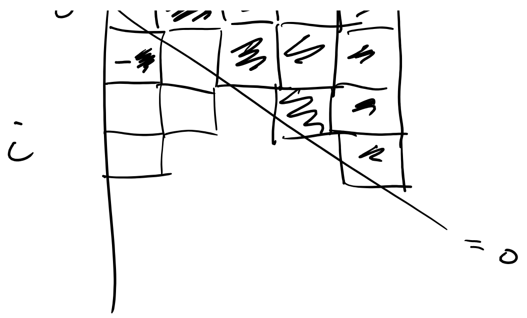




for (i = ...) a[i] = 0

for (i = 0 ; i < N ; ++i) {

for (j = i+1 ; j < N ; ++j) {

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for ( j=i+1; j<N; ++j){
    calculat the interaction a_ij
}
}
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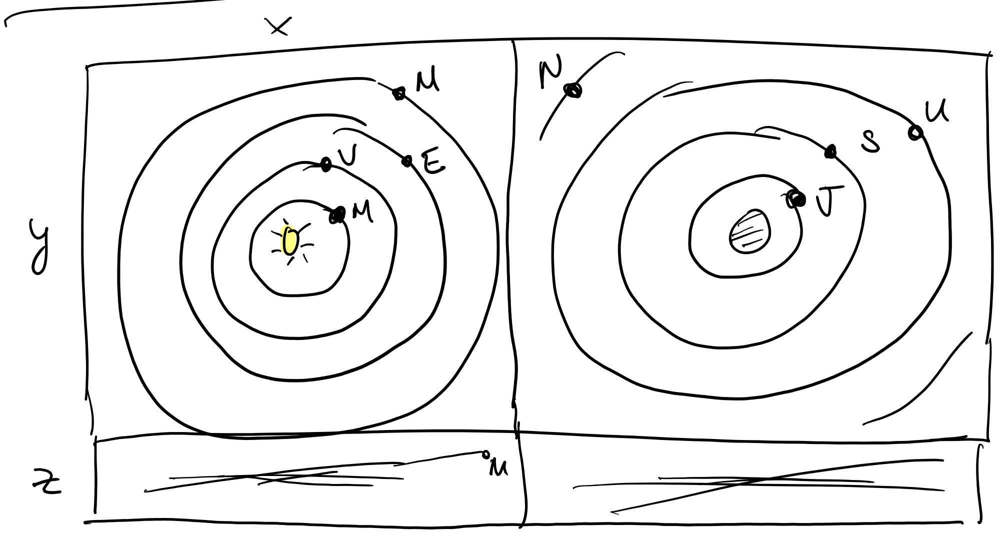
$$a[i] = a[i] + \frac{F}{m[i]}$$

$$a[j] = a[j] - \frac{F}{m[j]}$$



$\Delta t = 4$ days

We need some I.C.s in AU for $\underline{r}$ and $\frac{\text{A.U.}}{\text{day}}$ for $\underline{v}$

m in $M_\odot$

Julian Date = some number of days

Integrate ⟶ to "today"

in a file solar_data.dat