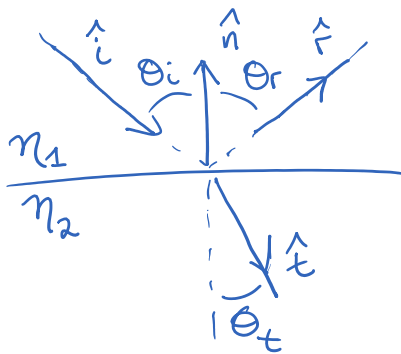


Reflection and Refraction

$$R + A = 1$$

$$R + T + \textcircled{A} = 1$$



Snell's Law

$$n_1 \sin \theta_i = n_2 \sin \theta_t$$

$$\sin \theta_t = \frac{n_1}{n_2} \sin \theta_i$$

$$\sin \theta_i > \frac{n_2}{n_1} \Rightarrow \text{TIR}$$

total internal reflection

$$t_{\perp} = t_{\parallel} + t_{\perp}$$

$$\boxed{T + R = 1}$$

FRESNEL EQUATION

$$R_{\perp}(\theta_i) = \left( \frac{n_1 \cos \theta_i - n_2 \cos \theta_t}{n_1 \cos \theta_i + n_2 \cos \theta_t} \right)^2$$

$$R_{\parallel}(\theta_i) = \left( \frac{n_2 \cos \theta_i - n_1 \cos \theta_t}{n_2 \cos \theta_i + n_1 \cos \theta_t} \right)^2$$

$$\sin^2 \theta_t = \left( \frac{n_1}{n_2} \right)^2 \sin^2 \theta_i = \left( \frac{n_1}{n_2} \right)^2 (1 - \cos^2 \theta_i)$$

$$\cos \theta_t = \sqrt{1 - \sin^2 \theta_t}$$

$$\rightarrow R(\theta_i) = \begin{cases} R_{\perp}(\theta_i) + R_{\parallel}(\theta_i) & \text{for no TIR} \\ 1 & \text{for TIR} \end{cases}$$

$R(\theta_i) = 1$  for TIR  
 $T(\theta_i) = 1 - R(\theta_i)$   
 can use this to modify your reflection for the opaque case.

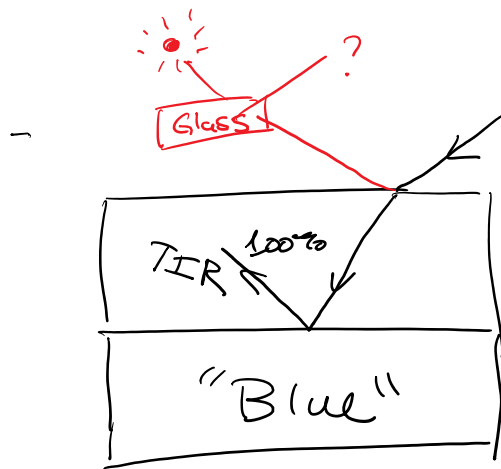
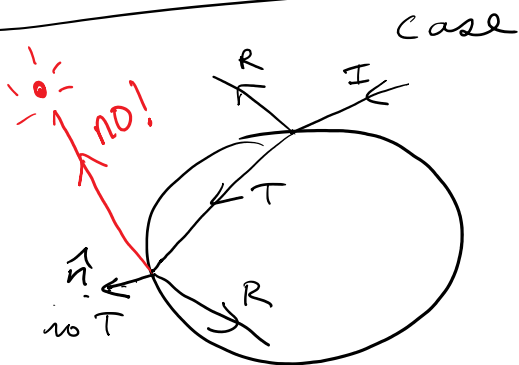
### Schlick's Approximation

$$R_{\text{sch}}(\theta_i) = R_0 + (1 - R_0)(1 - \cos\theta_i)^5$$

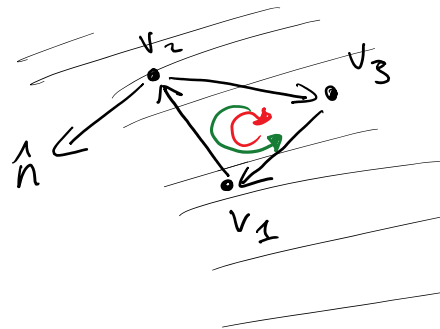
where  $R_0 = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$

careful:  $n_1 > n_2$   
 use  $\cos\theta_e$  instead of  $\cos\theta_i$

$$R_{\text{sch}} = \begin{cases} R_0 + (1 - R_0)(1 - \cos\theta_i)^5 & n_1 \leq n_2 \\ R_0 + (1 - R_0)(1 - \cos\theta_e)^5 & n_1 > n_2 \text{ and } \underline{\text{no TIR}} \\ 1 & n_1 > n_2 \text{ and TIR} \end{cases}$$



Intersection of Ray and triangle.



$$\hat{n} \cdot \underline{v}_i + \frac{D}{T} = 0$$

$$D = -\hat{n} \cdot \underline{v}_i$$

$$\underline{n} = (\underline{v}_2 - \underline{v}_1) \times (\underline{v}_3 - \underline{v}_1)$$

$$\underline{r} = \underline{d}t + \underline{o}$$

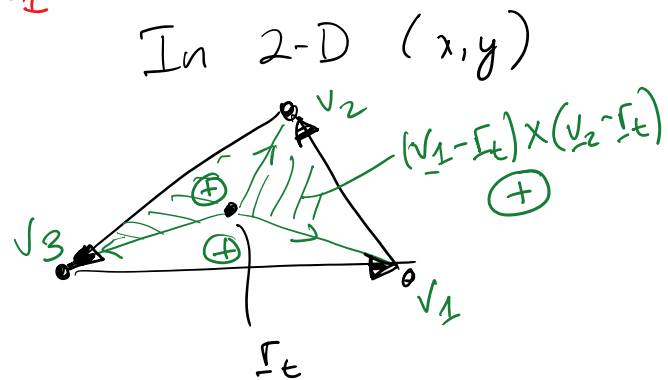
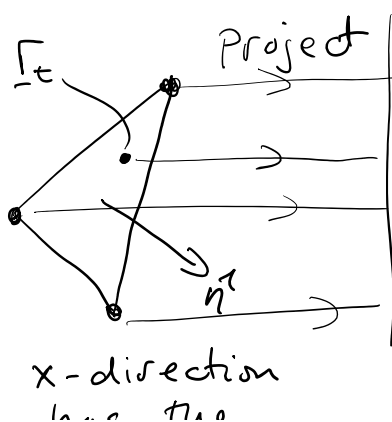
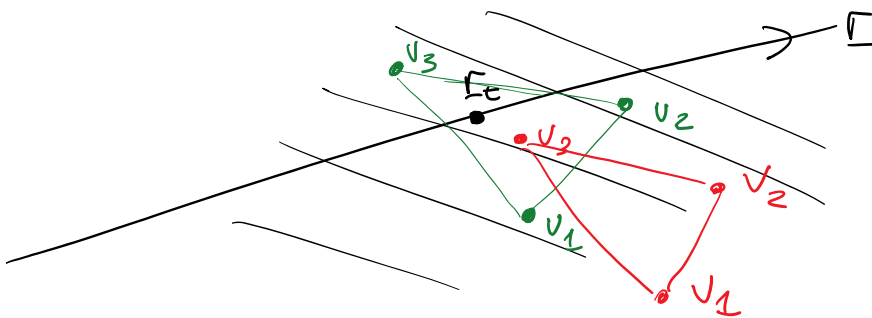
Put this into the plane equation and solve for  $t$ :

$$\hat{n} \cdot \underline{r} + D = 0$$

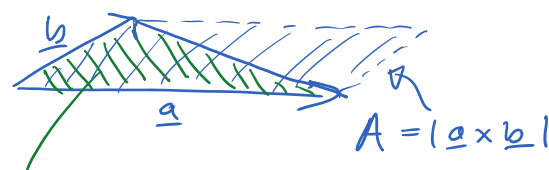
$$(\hat{n} \cdot \underline{d})t + (\hat{n} \cdot \underline{o}) + D = 0$$

$$t = \frac{-(\hat{n} \cdot \underline{o}) - D}{(\hat{n} \cdot \underline{d})} \quad \text{! what if } = 0!?$$

$\underline{r}_t$  is a possible point in the triangle.

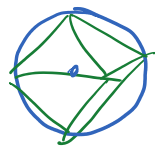
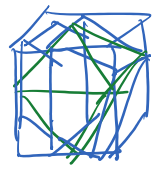
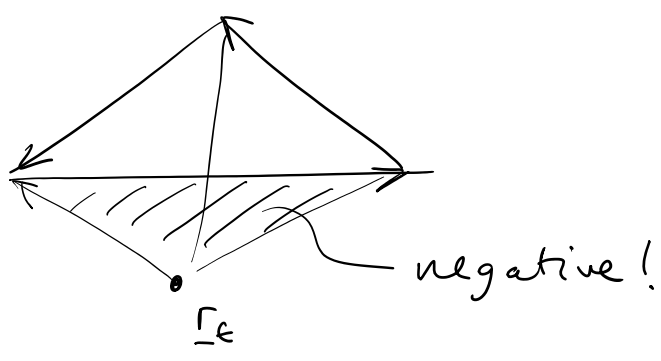


x-direction  
has the  
largest component  
of  $\hat{n}$



$$A_T = \frac{1}{2} |a \times b|$$

All 3 subtriangles are  $\oplus$  cross product  
if  $r_t$  is inside the triangle  
or all  $\ominus \Rightarrow r_t$  inside



Bounding Volume  
Hierarchy — "TREE"