Simulations in the Natural Sciences

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Evaluation: 60% ORAL / 40% Exercises!

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0: 000000000000 — 32bit, 64bit

−1: 11111 11

2s Complement

Most negative: $1\underbrace{0000\ldots000}_{31\times}$ $\quad -2^{31}$

Most positive: $0\underbrace{111\ldots\ldots 1}_{31\times}$ $\quad +2^{31}-1$

Floating point is not Real Numbers

1985 IEEE-754

32 bit: float

$\pm 1.23456 \times 10^{12}$

$\underbrace{\boxed{\pm}}_{S}\ 1.\underbrace{\boxed{01011101}}_{\text{Mantissa}} \times 2^{\boxed{101}}$ ← Exponent

S	8x E	M 23x

0

$s \times 1.M \times 2^{E-127}$

1.65 → 1.7

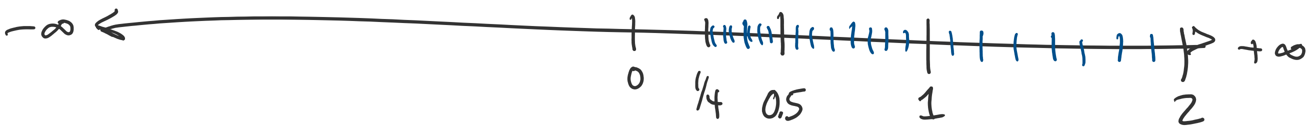
1.75 → 1.8

1.6 | 9, 8, 7, 6 } 1.7 ; 5 ; 4, 3, 2, 1 } 1.6

Round to nearest even

```
r2 = x*x + y*y + z*z;
assert(r2 >= 0);
r = sqrt(r2);
```

NaN



$$f(x) = 0 \qquad ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \boxed{A}$$

$$x = \frac{2c}{-b \pm \sqrt{b^2 - 4ac}} \qquad \boxed{B}$$

If either of these are used you are asking for trouble! Round-off

- when a and/or c are very small and $b \geq 0$

$$-b \pm \sqrt{b^2 - \text{very small}} \qquad \text{JUST DON'T}$$

$$q = -\frac{1}{2}\left[b + \text{sign}(b)\sqrt{b^2 - 4ac} \right]$$

$$x_1 = \frac{q}{a} \qquad x_2 = \frac{c}{q}$$

are the 2 roots.

<u>Stability</u>: Example: calculate the integer powers of ϕ (Golden Mean)

$$\phi = \frac{\sqrt{5} - 1}{2} \approx 0.61803\ldots$$

$$\phi^n : = \prod^n \phi \qquad \phi^{n+1} = \phi^{n-1} - \phi^n$$

$$\phi^n := \prod_{i=1}^{n} \phi \qquad \boxed{\phi^{n+1} = \phi^{n-1} - \phi^n}$$

$$\phi^0 = 1 \qquad \phi^1 = \frac{\sqrt{5}-1}{2}$$

Another solution $\phi^+ \equiv -\frac{\sqrt{5}+1}{2}$

$$\phi^1 = \underbrace{\phi + \varepsilon \phi^+}_{\text{linear combination}} \qquad |\phi^+| > 1$$