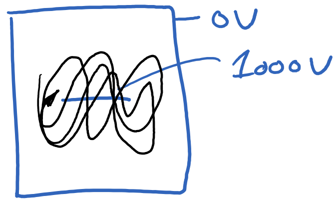
$$\underline{F} = m\,\underline{a}$$



Electrodynamics

$$\underline{F} = e(\underline{E} + \cancel{v_e \times \underline{B}}) \quad \cancel{v_e \times \underline{B}} \to \emptyset$$

$$\underline{E} = -\nabla\Phi$$

$$\dot{x} = V_x$$

$$\underline{a} = \left(\frac{e}{m_e}\right)(-\nabla\Phi)$$

$$\dot{V}_x = \ddot{x} = \left(\frac{e}{m_e}\right)\left(-\frac{\partial\Phi}{\partial x}\right)$$

$$\ddot{y} = \left(\frac{e}{m_e}\right)\left(-\frac{\partial\Phi}{\partial y}\right)$$

$$e = 1.60 \times 10^{-19}\ C$$

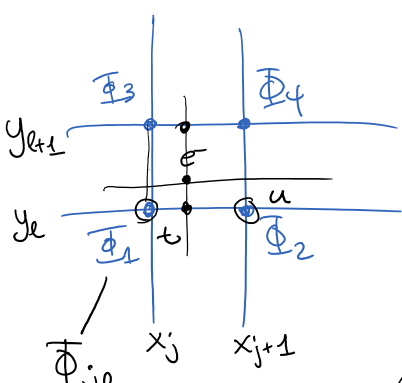
$$m_e = 9.11 \times 10^{-31}\ kg$$

$$\left(\frac{e}{m_e}\right) = 1.756 \times 10^{11}\ \frac{C}{kg}$$

$$\Phi = \left[\frac{Nm}{C}\right] \qquad \nabla\Phi = \left[\frac{N\cancel{m}}{C\cancel{m}}\right]$$

$$\left(\frac{e}{m_e}\right)(-\nabla\Phi) = \left[\frac{\cancel{C}}{kg}\right]\left[\frac{N}{\cancel{C}}\right] = \left[\frac{N}{kg}\right] = \left[\frac{\cancel{kg}\,m s^{-2}}{\cancel{kg}}\right]$$

$$\ddot{x} = \left[\frac{m}{s^2}\right]$$



$$t = \frac{(x - x_j)}{(x_{j+1} - x_j)} = \boxed{\frac{1}{\Delta}(x - x_j)}$$

$$u = \frac{1}{\Delta}(y - y_l) \qquad \frac{\partial t}{\partial x} = \frac{1}{\Delta}$$

$$\Phi(x, y_l) = (1-t)\Phi_1 + t\Phi_2$$

$$\Phi(x_j, y) = (1-u)\Phi_1 + u\Phi_3$$

$$\Phi(x, y_{l+1}) = (1-t)\Phi_3 + t\Phi_4$$

$$\boxed{\Phi(x,y) = (1-t)(1-u)\Phi_1 + (1-u)t\Phi_2 + u(1-t)\Phi_3 + ut\Phi_4}$$

Bilinear interpolation

Bilinear interpolation

$$\left.\frac{\partial \Phi}{\partial x}\right|_u = \frac{\partial t}{\partial x}\frac{\partial \Phi}{\partial t} = \frac{1}{\Delta}\left[-(1-u)\Phi_1 + (1-u)\Phi_2 - u\Phi_3 + u\Phi_4\right]$$

$$= \frac{1}{\Delta}\left[(1-u)(\Phi_2 - \Phi_1) + u(\Phi_4 - \Phi_3)\right]$$

$$\left.\frac{\partial \Phi}{\partial y}\right|_t = \text{Exercise} \ldots$$

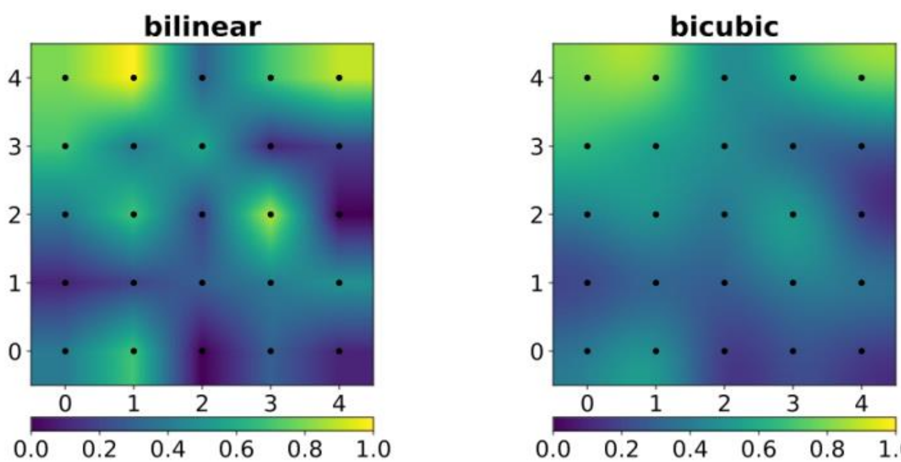
$$\dot{x} = V_x$$

$$\dot{V}_x = \left(\frac{e}{m_e}\right)\left(-\frac{1}{\Delta}\left[(1-u)(\Phi_2 - \Phi_1) + u(\Phi_4 - \Phi_3)\right]\right)$$

$$\dot{y} = V_y$$

$$\dot{V}_y = \left(\frac{e}{m_e}\right)\left(-\frac{1}{\Delta}\left[\cdots\right]\right)$$

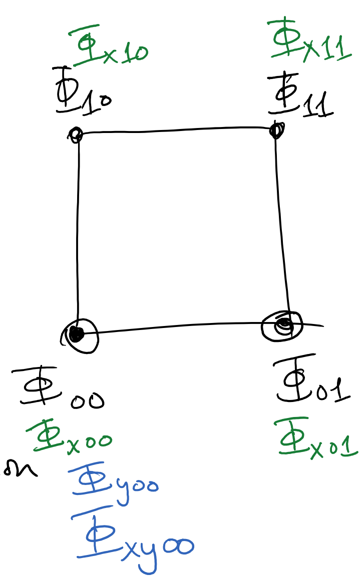
Leapfrog or RK2/4



$$\boxed{\Phi(x,y) = \sum_{i=0}^{3}\sum_{j=0}^{3} a_{ij}\, x^i y^j}$$

16 coefficients

16 equations



$$\Phi_{00} = \Phi(0,0) = \text{ when } x^0 = y^0 = 1$$

$$= a_{00} \text{ gives a contribution}$$

$$\Phi_{01} = \Phi(1,0) = \sum^{3} a_{i0}$$

Φ_{xy00}

$$\Phi_{01} = \Phi(1,0) = \sum_{i=0}^{3} a_{i0}$$

$$\Phi_{10} = \Phi(0,1) = \sum_{j=0}^{3} a_{0j}$$

$$\Phi_{11} = \Phi(1,1) = \sum_{i=0}^{3} \sum_{j=0}^{3} a_{ij}$$

$$\Phi_{x00} = \Phi_x(0,0) = \sum_{i=0}^{3} \sum_{j=0}^{3} a_{ij} \left(i x^{i-1} \right) y^j$$

$$= a_{10}$$

$$\Phi_{x01} = \Phi_x(1,0) = \sum_{i=1}^{3} a_{i0} \cdot i$$

$$\Phi_{x10} = \sum_{j=0}^{3} a_{1j}$$

$$\Phi_{x11} = \sum_{i=1}^{3} \sum_{j=0}^{3} a_{ij} \cdot i$$

$$x = \overbrace{A}^{16 \times 16} \alpha$$

$$x = [\Phi_{00} \ \Phi_{10} \ \Phi_{01} \ \Phi_{11} \ \Phi_{x00} \ \Phi_{x10} \ \Phi_{x01} \ \Phi_{x11} \ \cdots]$$

16 length vector

$$\alpha = [a_{00} \ a_{10} \ a_{20} \ a_{30} \ a_{01} \ a_{11} \ \cdots]$$

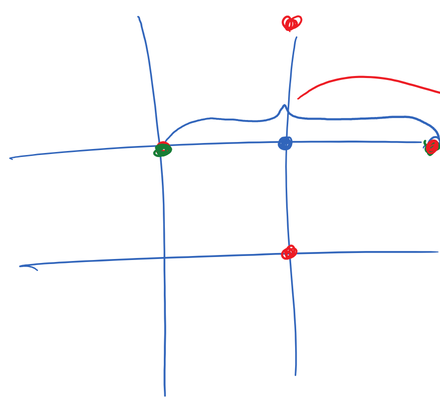
$$\Rightarrow \alpha = A^{-1} x$$

$$\begin{bmatrix} a_{00} & a_{01} & a_{02} & a_{03} \\ a_{10} & a_{11} & a_{12} & a_{13} \\ a_{20} & a_{21} & a_{22} & a_{23} \\ a_{30} & a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -3 & 3 & -2 & -1 \\ 2 & -2 & 1 & 1 \end{bmatrix} \cdot$$

$$\begin{bmatrix} \Phi(0,0) & \Phi(0,1) & \Phi_y(0,0) & \Phi_y(0,1) \\ \Phi(1,0) & \Phi(1,1) & \Phi_y(1,0) & \Phi_y(1,1) \\ \Phi_x(0,0) & \Phi_x(0,1) & \Phi_{xy}(0,0) & \Phi_{xy}(0,1) \\ \Phi_x(1,0) & \Phi_x(1,1) & \Phi_{xy}(1,0) & \Phi_{xy}(1,1) \end{bmatrix} \cdot$$

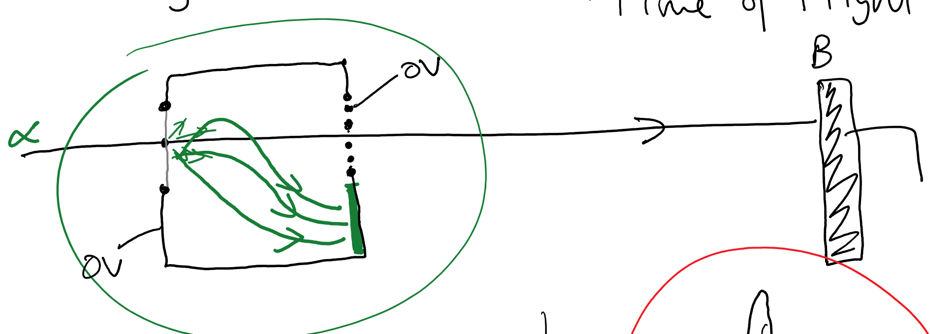
$$\begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 0 & 3 & -2 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

$$\Phi(x,y) = \begin{bmatrix} 1 & x & x^2 & x^3 \end{bmatrix} \cdot \begin{bmatrix} a_{ij} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ y \\ y^2 \\ y^3 \end{bmatrix}$$

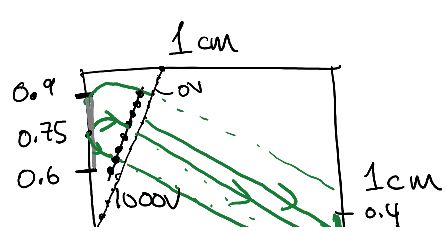


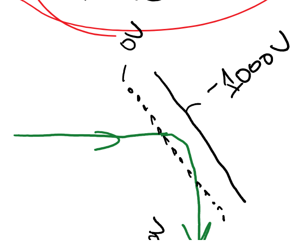
$$\Phi_x(1,1) = \frac{1}{2\Delta}\left(\Phi_{j+2,l+1} - \Phi_{j,l+1}\right)$$

The Design Prize! "Time of Flight"

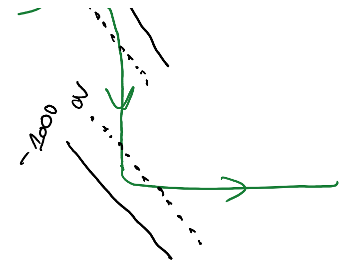


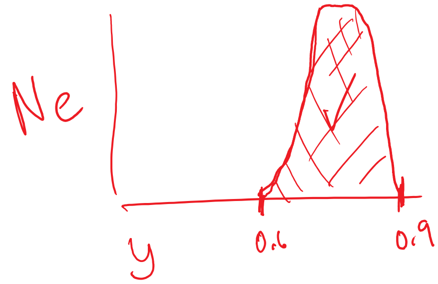
$|V_e| = 10^6$ m/s

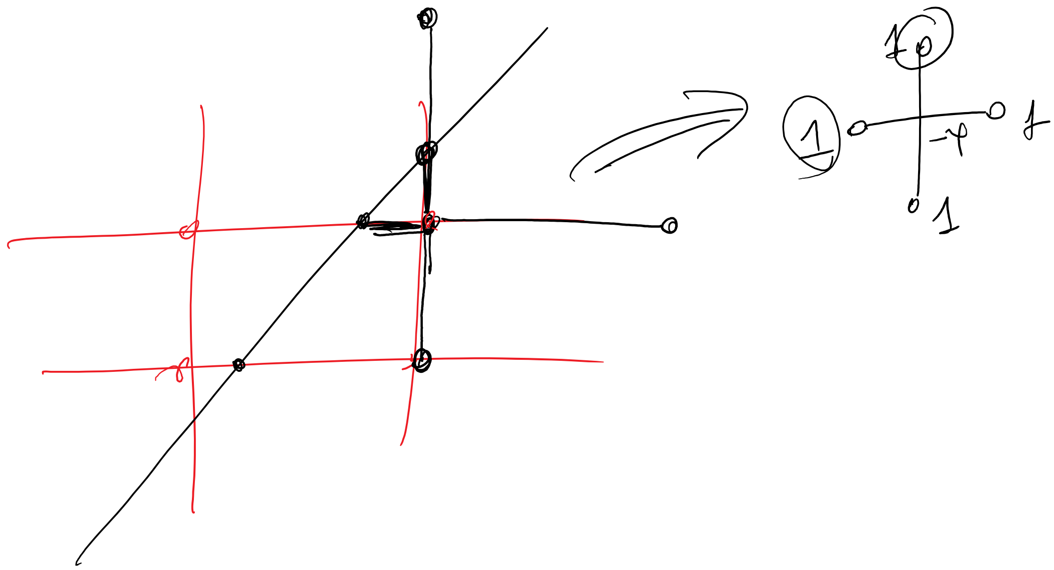





0.75
0.6
1000V
1cm
0.4
0.1
0


-1000 V


Ne
y
0.6
0.9


1
1
-4
1
1