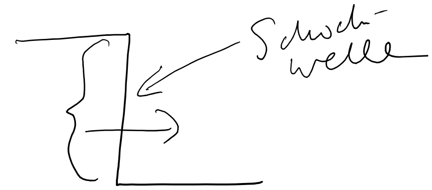
Spin 1 - Finite Volume und das Riemann Problem

Monday, May 30, 2016 1:00 PM

Finite Difference : $\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = 0$

Erhaltungsgleichungen $\longrightarrow$



Diskontinuitäten

Masse, Momentum, Energie
(1) (3) (1)

Integral-gleichungen

Rankine-Hugnoit Konditionen

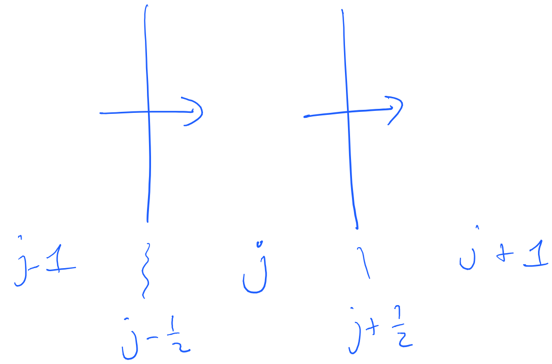
Grösse und Geschwindigkeit der Diskontinuitäten.

☆ Erhaltung sollte eine Eigenschaft der Numerischen Methode sein.

$$u_j^{(n+1)} = u_j^{(n)} + \frac{\Delta t}{\Delta x}\left[f_{j-\frac{1}{2}} - f_{j+\frac{1}{2}}\right]$$

Numerischen Fluss

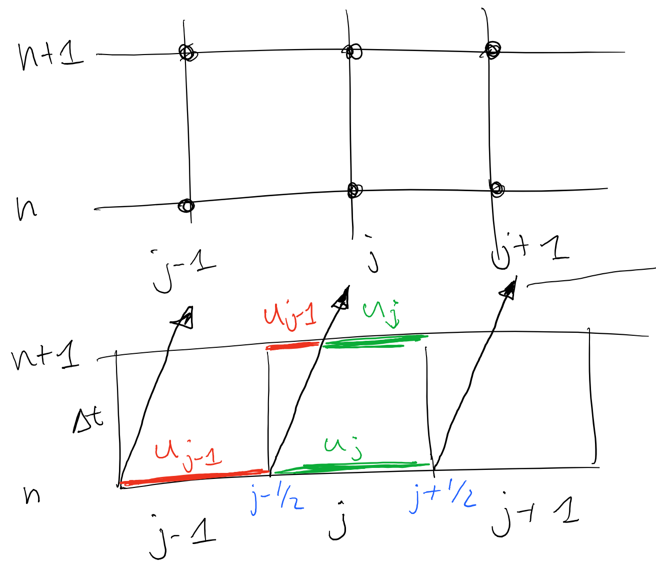
F's dürfen eine Approximation sein, aber die Integration dieses Flusses über der Zeit ist <u>Exact</u>.



$$\frac{\partial u}{\partial t} + \frac{\partial f(u)}{\partial x} = 0$$

$$\boxed{\begin{array}{c} f(u) = au \\ \text{für Linear Advection} \end{array}}$$

"Finite Volume" Methoden



"Finite Difference"

Characteristic Neigung a für unser einfaches Prob.

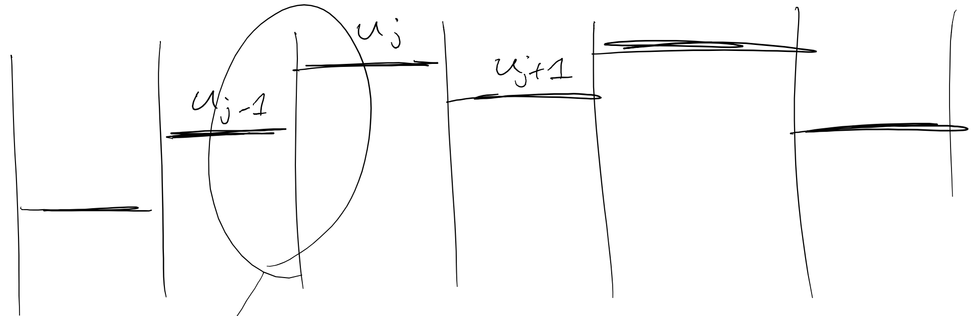
Godunov

$$u_j^{(n+1)} = \underbrace{\frac{a \cdot \Delta t}{\Delta x}}_{c} u_{j-1}^{(n)} + \left(1 - \frac{a \cdot \Delta t}{\Delta x}\right) u_j^{(n)}$$

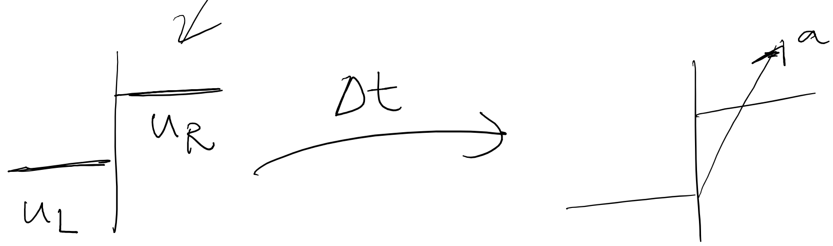
$$u_j^{(n+1)} = c\, u_{j-1}^{(n)} + (1-c)\, u_j^{(n)}$$

$$u_j^{(n+1)} = u_j^{(n)} + c\left(u_{j-1}^{(n)} - u_j^{(n)}\right)$$

CIR upwind schema.



Stückweise Konstant



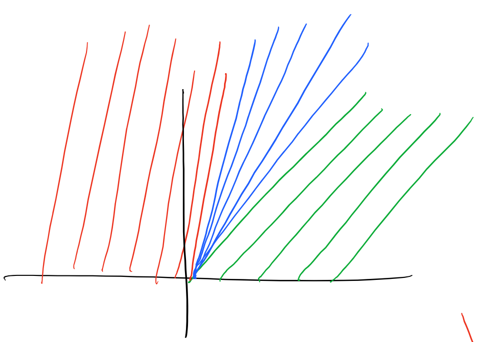
Riemann Problem

$$f' = RP_{\Delta t}(u_L, u_R)$$

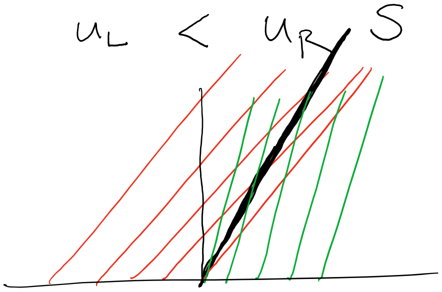
Inviscid Burger's Equation $f(u) = \frac{1}{2}u^2$

$$\frac{\partial u}{\partial t} + \frac{\partial \frac{1}{2}u^2}{\partial x} = 0$$

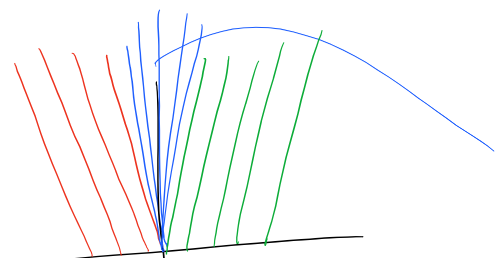
$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0$$



$u_L < u_R$



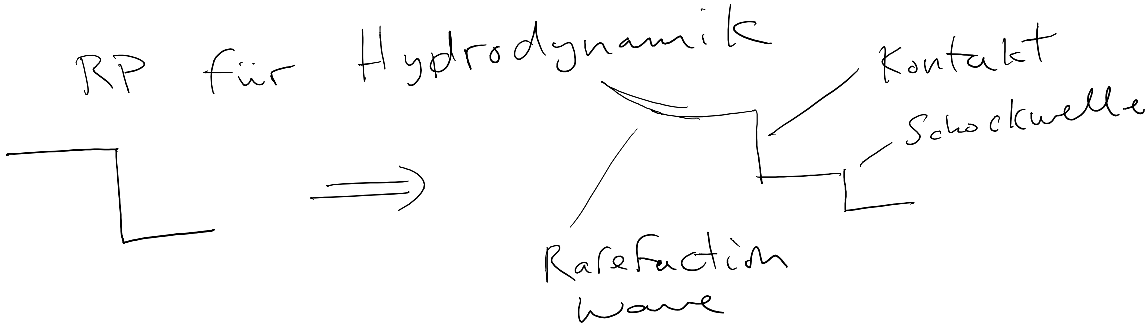
$u_L > u_R$



Sonic Point

Fächer / Fan

RP für Hydrodynamik



$$u_j^{(n+1)} = \frac{1}{\Delta x} \int_{x_{j-1/2}}^{x_{j+1/2}} \tilde{u}(x, \Delta t)\, dx$$

wird bestimmt durch das Lösen von zwei Riemann Problemen

$$RP(u_{j-1}^{(n)}, u_j^{(n)})$$

$$RP(u_j^{(n)}, u_{j+1}^{(n)})$$