Constant Growth rate for the population:

$$\frac{P_{n+1} - P_n}{P_n} \equiv r$$

or

$$\boxed{P_{n+1} = (1+r) P_n}$$

There is a solution for all n and for all r,

$$P_{n+1} = (1+r)^n P_0$$

Population Explosion!

Normalize the population with respect to some "maximal" size N that the resources can support:

$$\boxed{p = P/N}$$

p	r
1	0
small	large and positive
~ 1	small
>1	negative

$$r \propto (1-p)$$

$$r = k(1-p)$$

Verhulst

$$\frac{p_{n+1} - p_n}{p_n} = k(1-p_n)$$

$$p_{n+1} = p_n + k \boxed{p_n} (1 - \boxed{p_n})$$

Deterministic!

Non Linear System

Non-linear System

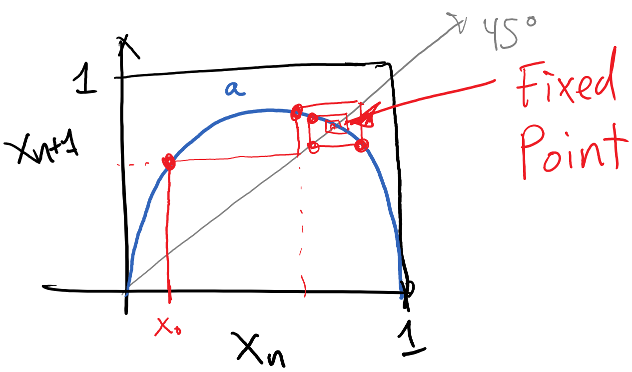
Eg, $K = 3$ $\quad p_0 = 0.01$

$p_0' = 0.00999999999\ldots$

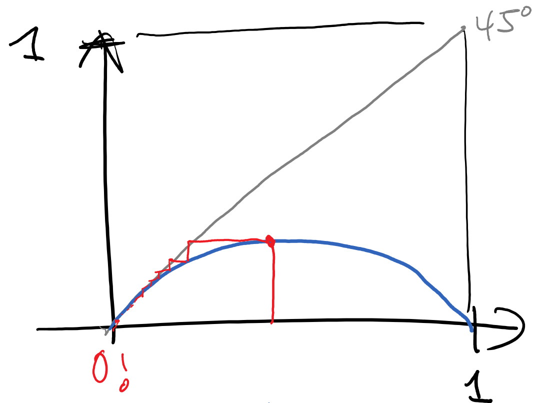
$$x_{n+1} = a\, x_n (1 - x_n) \qquad \text{Logistic Equation}$$

$$x \in [0, 1]$$

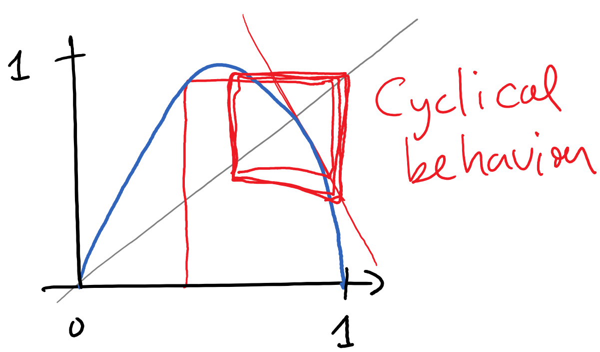
$$a \in [0, 4]$$



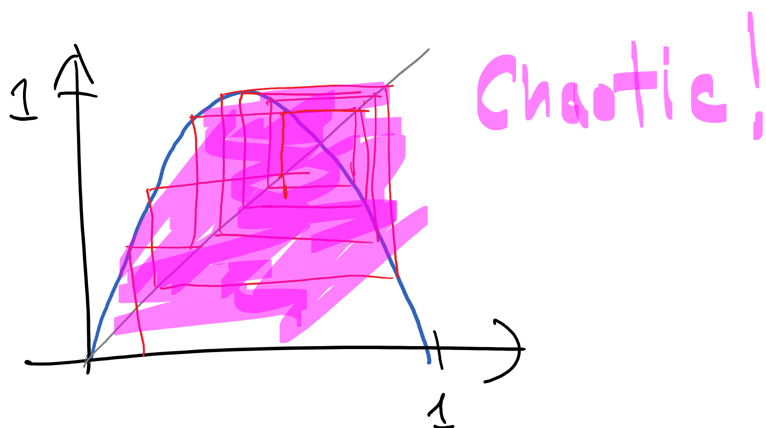
$a = 2.75$



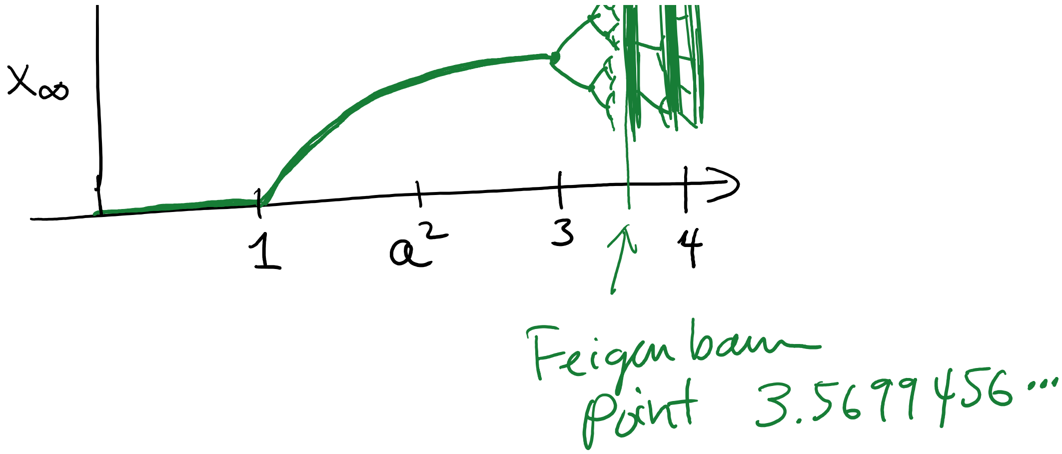
$a = 1.45$



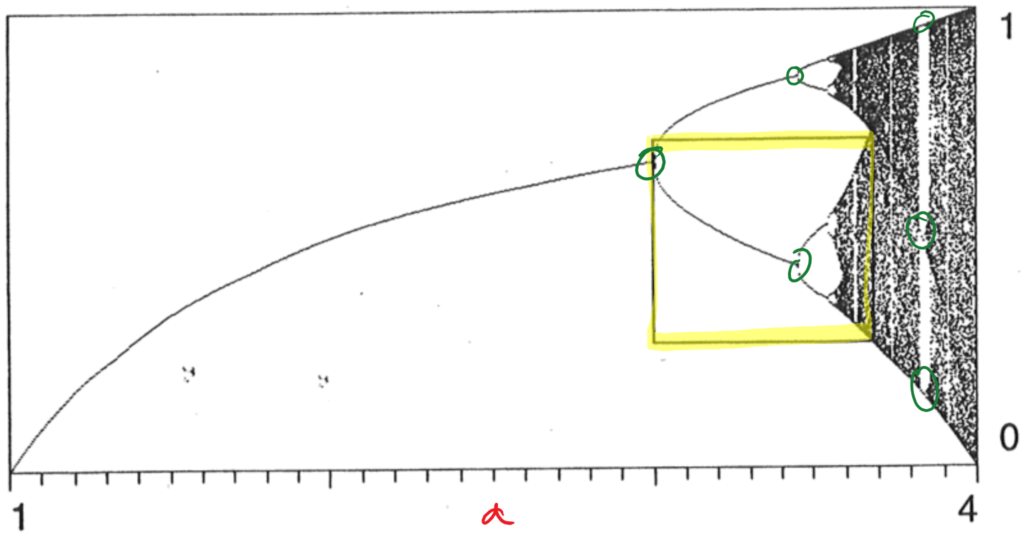
$a = 3.2$



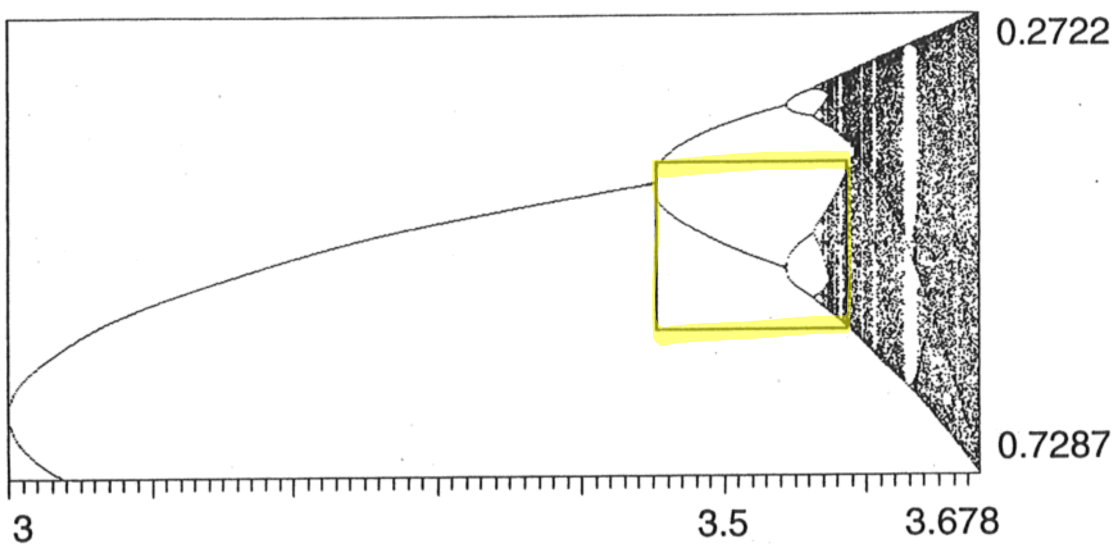
Feigenbaum Diagram



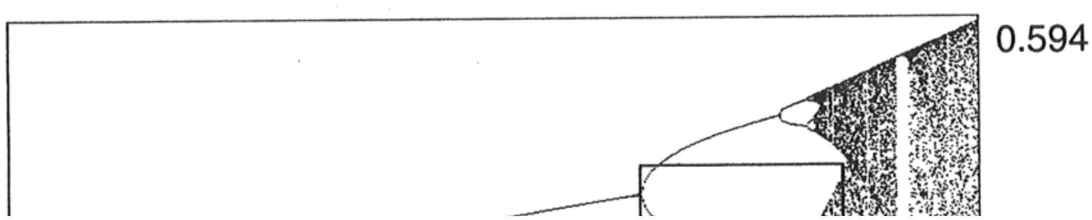
X_∞
1
a^2
3
4
Feigenbaum
Point 3.5699456...



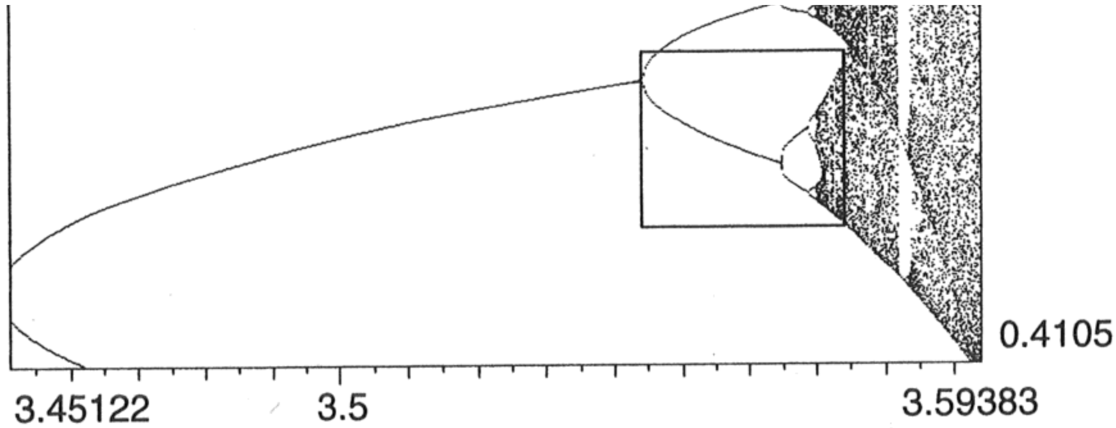
1
0
1
a
4

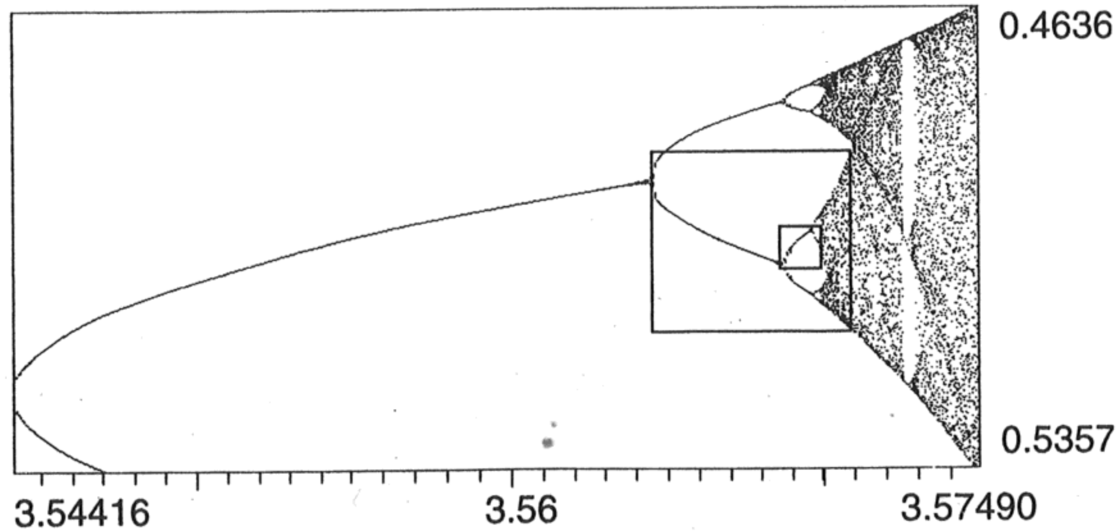


0.2722
0.7287
3
3.5
3.678

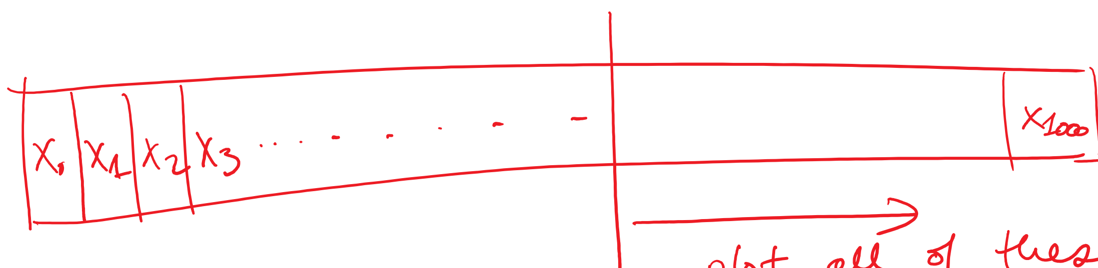


0.594

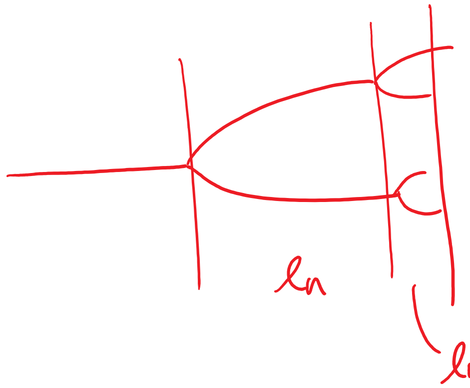




FRACTAL



plot all of these values for a given a.



$$\frac{l_n}{l_{n+1}} = 4.6692\ldots$$

Universal Property!