Fractals using complex numbers

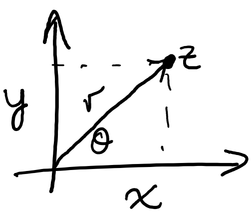
05 October 2020 12:35

Non-linearity : $x_n(1-x_n)$

Complex Iterators : $i^2 = -1$

$$z = (x + iy)$$

Real Imaginary



$$z = re^{i\theta}$$

let $z = x + iy$

$w = u + iv$

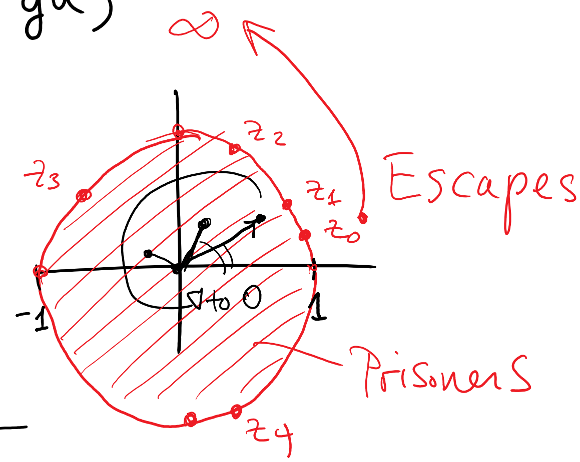
$$z \cdot w = (x+iy)(u+iv)$$
$$= xu + ixv + iyu - yv$$
$$= (xu - yv) + i(xv + yu)$$

$$z^2 = (re^{i\theta})^2 = r^2 e^{i(2\theta)}$$

$$z_{n+1} = z_n^2$$

Call this set the Julia Set

"boring"



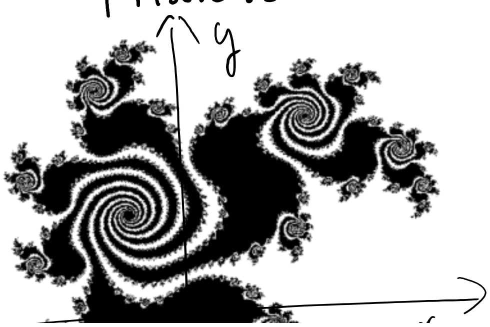
$$z_{n+1} = z_n^2 + C$$

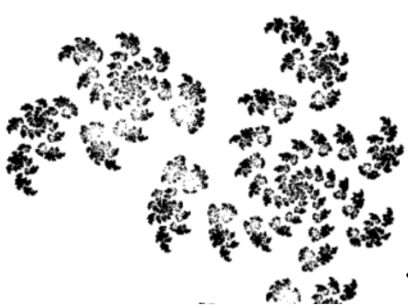
← add a complex constant.

What is the Julia Set of this?

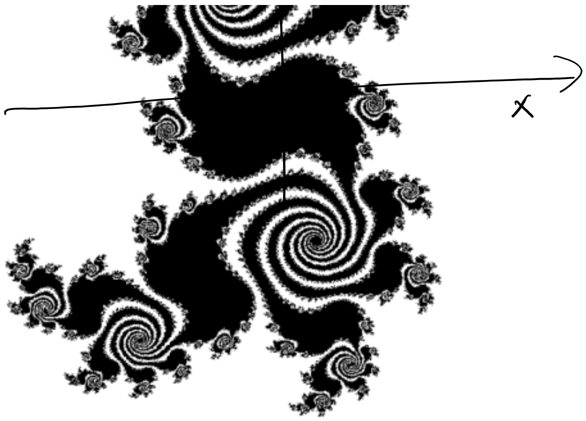
Fractal Prison

Prisoners





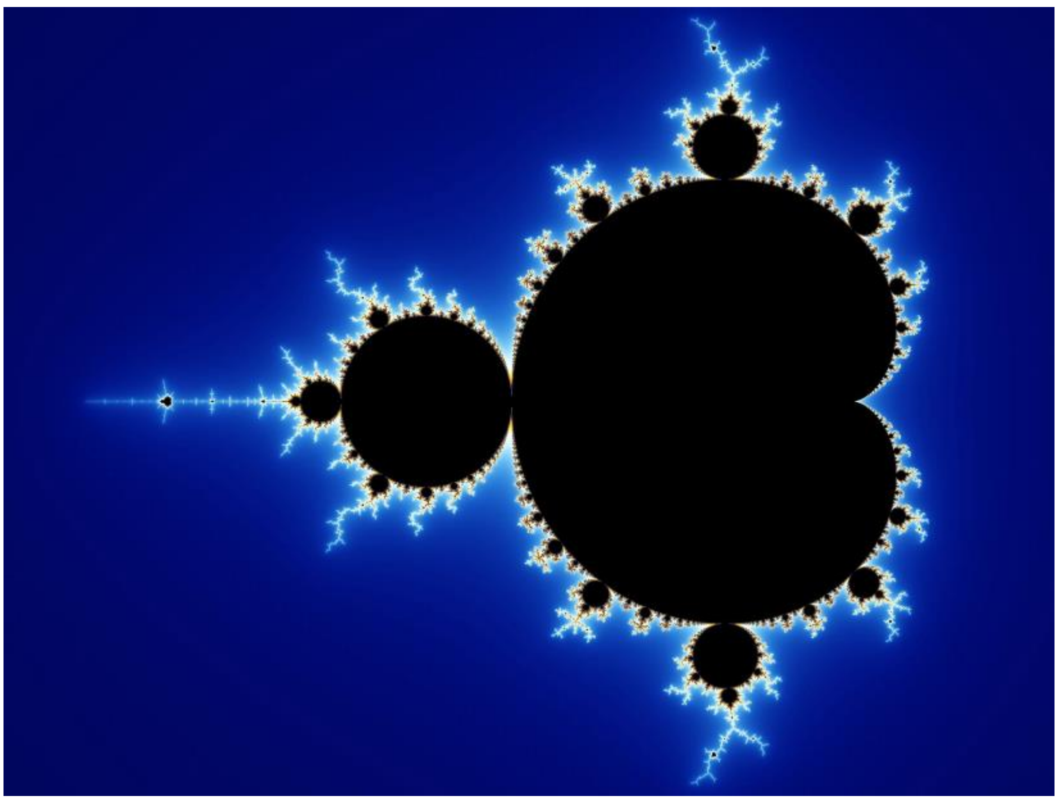
← Dust



For $c = -0.5 + 0.5i$
A connected J_c.

← Dust

Unconnected
for some other c.
unconnected J_c.



Mandelbrot Set : Black is J_c connected
Colored is J_c unconnected

$$M = \{ c \in \mathbb{C} \mid J_c \text{ is connected} \}$$

Another definition

$$M = \{ z_0 = c \in \mathbb{C} \mid z_{n+1} = z_n^2 + c < \infty \}$$

If $|z_n| > r(c)$ a critical radius, then it will always escape to infinity!

$r(c) := \max(|c|, 2)$

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colors: according to how many steps to escape it takes. How "close" to the Mandelbrot set is it?

Do for all c (pixels on the screen)

```
k = 0
z = c
while (k < 100) {
    if (|z| > r(c)) {
      Draw point with color(k);
      return (c ∉ M)
    }
    z = z*z + c
    k = k+1
}
Draw point with color("Black");
return (c ∈ M)
```

In population growth: $\dfrac{P_{n+1} - P_n}{P_n} = r$ $\quad \dfrac{1}{P}\dfrac{dP}{dt} = r$

Differential Equations:

ODE: ordinary differential equations

PDE: partial differential equations (later in the course)

$$\frac{d^2 y}{d x^2} + q(x)\frac{dy}{dx} = r(x)$$: 2^{nd} order $\dfrac{d^2}{dx^2}$

du $\quad$ 2×1^{st} order

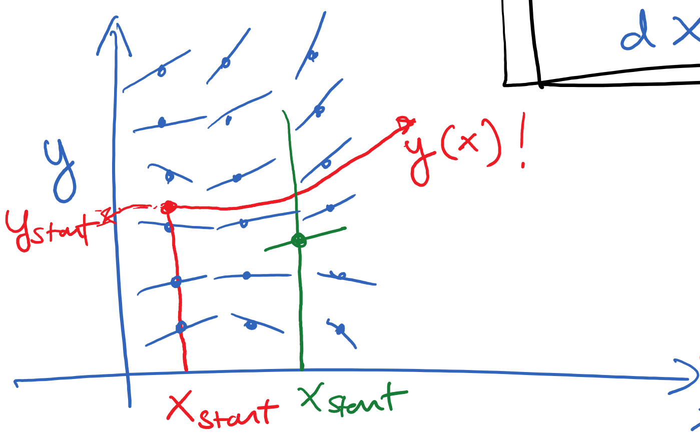
$$\boxed{\begin{aligned} \frac{dy}{dx} &= z(x) \\ \frac{dz}{dx} + \boxed{q(x)\cdot z(x)} &= r(x) \end{aligned}}$$

2×1^{ST} order equations. ✓

In general: $\dfrac{dy_i(x)}{dx} = f_i(x, y_0(x), y_1(x), \dots, y_{N-1}(x))$

$i = 0 .. N-1$

$$\boxed{\frac{d\bar{y}(x)}{dx} = \bar{f}(x, \bar{y})}$$



$\dfrac{dy}{dx} = f(x,y)$

Need an initial condition - time

Boundary conditions

$y_i(x_{start}) = y_i^{START} \leftarrow$ given

(Dirchlet Boundary conditions)

$\dfrac{dy_i}{dx}(x_{start}) =$ Slope is given somewhere

(von Neumann Boundary condition)

Analytical solutions are difficult.

Numerical solutions (algorithm) is easy!

↳ Numerical errors need to be kept under control/tracked.