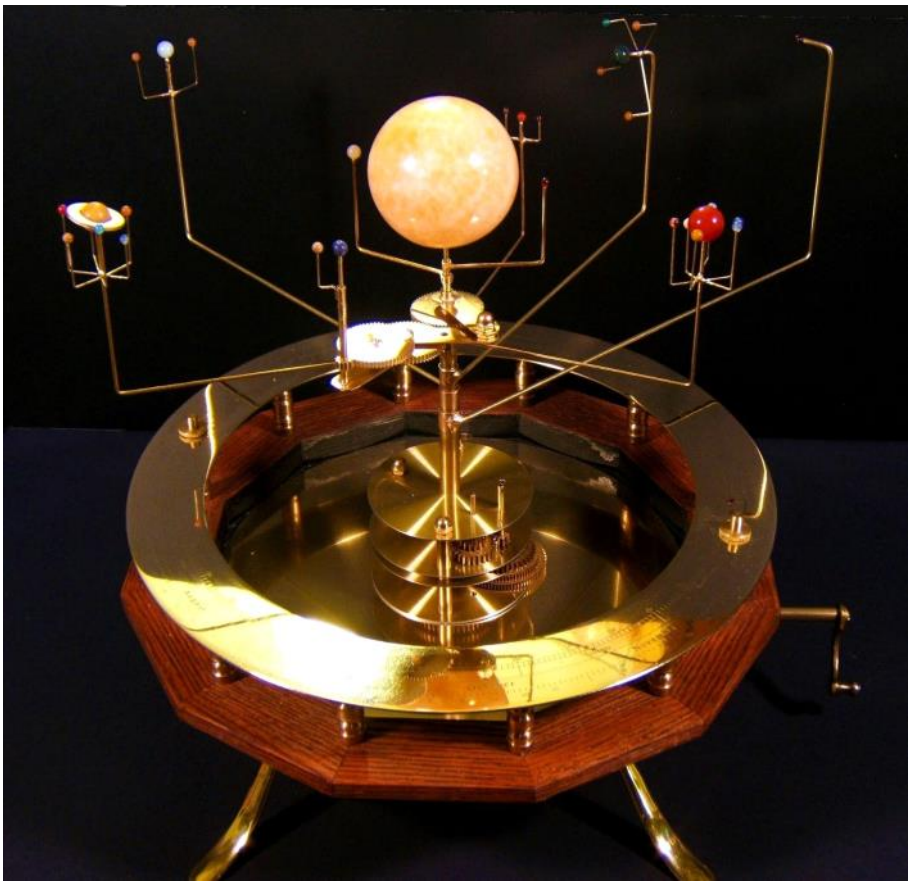
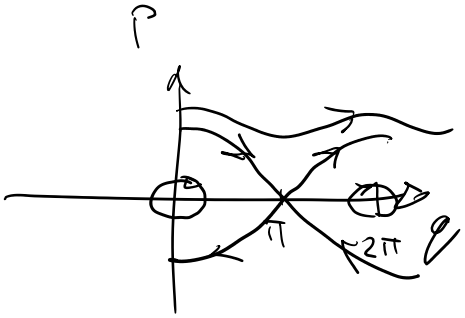
N-body Problem, Simulating the Solar System

Monday, 30 October 2023 11:29



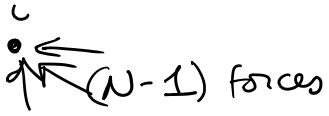


8 Planets + Sun → 9 bodies

$H = T + U$

Kinetic Potential

How many Forces? $N = 9$



$N \cdot (N-1)$ Forces

$F_{ij} = -F_{ji}$ Newton's 3rd Law

$\frac{N(N-1)}{2}$ interactions

$O(N^2)$

Imagine how many interactions for $N = 10^{12}$ bodies. Possible?

$N = 10^{12}$ bodies. Possible?

$N_{interactions} = \frac{10^{24}}{2} \cdot 20$ Flops

$N_{FLOPS} = 10^{25}$

How many calculations?

$\frac{1}{d} \equiv \frac{1}{\sqrt{d^2}}$

$O(20)$

$10^{7.5} = 1$ yr.

Giga $= 10^9 s^{-1}$

Tera $= 10^{12} s^{-1}$

Peta $= 10^{15} s^{-1}$

Exa $= 10^{18} s^{-1} \Longrightarrow \frac{10^{25}}{10^{18}} = 10^7 s \cong \frac{1}{3}$ year

Aside

$O(N^2) \Rightarrow O(N \log N) \Rightarrow O(N)$

$$\underline{F}_{ij} = \frac{G m_i m_j}{|\underline{r}_i - \underline{r}_j|^2} \frac{(\underline{r}_j - \underline{r}_i)}{|\underline{r}_j - \underline{r}_i|}$$

$$\underline{F}_{ij} = \frac{G m_i m_j}{|\underline{r}_j - \underline{r}_i|^3} \cdot (\underline{r}_j - \underline{r}_i)$$

$$\underline{F}_i = \sum_{j \neq i} \frac{G m_i m_j}{|\underline{r}_j - \underline{r}_i|^3} (\underline{r}_j - \underline{r}_i)$$

G = 5 decimal places

$GM_\odot = k^2$ Gauss' Gravitational Const
15 decimal places

$$\underline{F}_i = \sum_{j \neq i} \frac{k^2 m_i m_j}{|\underline{r}_j - \underline{r}_i|^3} (\underline{r}_j - \underline{r}_i)$$

$m_i \; [M_\odot]$

$$\dot{p} = -\frac{\partial H}{\partial q} = -\frac{\partial U(q)}{\partial q}$$

m_i [$M_\odot$]

r_i [AU]

t [days]

$$P = -\frac{\partial}{\partial q} = -\frac{\partial}{\partial q} = -\nabla U$$

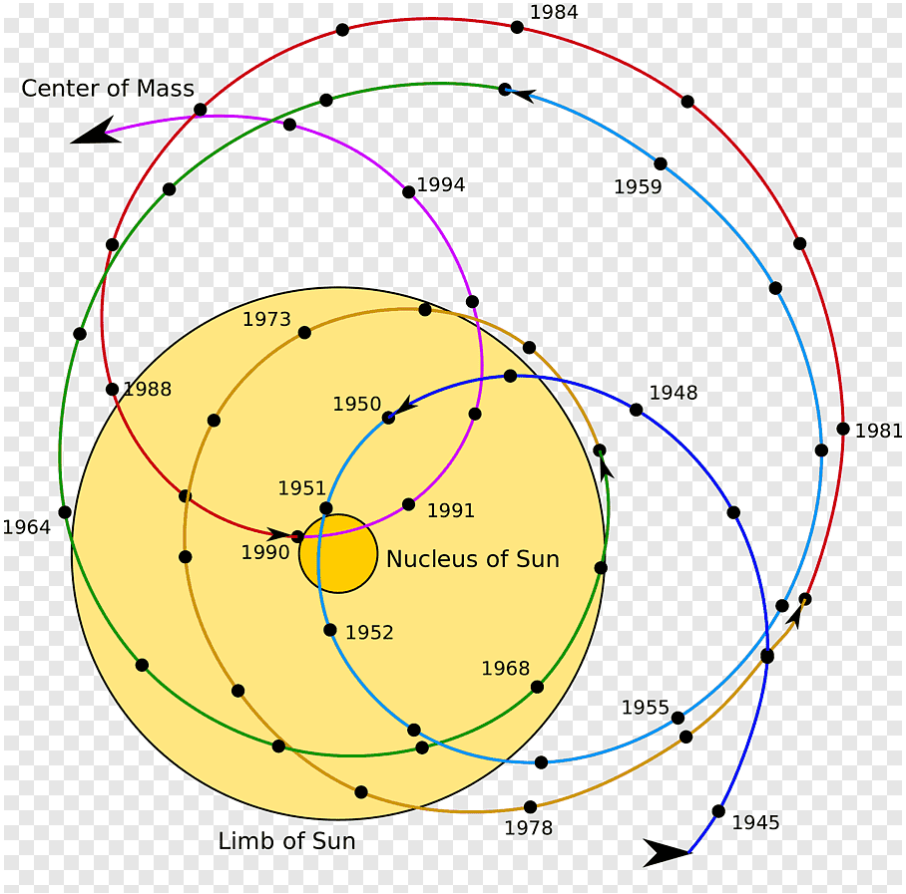
$$a_i = \frac{F_i}{m_i} = -\nabla U$$

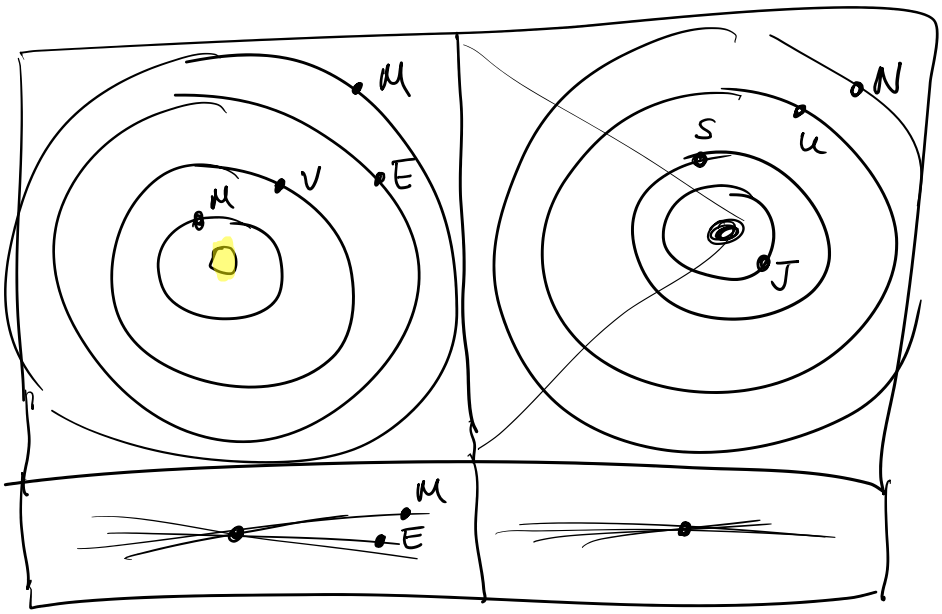
Leapfrog:

Drift (H=T) $\forall i$ $\quad \underline{r}_{i,n+\frac{1}{2}} = \underline{r}_{i,n} + \frac{h}{2}\underline{v}_{i,n}$

Kick (H=U) $\forall i$ $\quad \underline{v}_{i,n+1} = \underline{v}_{i,n} + h\,\underline{a}_i\left(\{\underline{r}_{i,n+\frac{1}{2}}\}\right)$

Drift (H=T) $\quad \underline{r}_{i,n+1} = \underline{r}_{i,n+\frac{1}{2}} + \frac{h}{2}\underline{v}_{i,n+1}$





The IC?

file solar_data.dat

$\underline{r}$ in [AU] $\underline{v}$ in $\left[\frac{AU}{days}\right]$

m in $[M_\odot]$

t_{IC} = Julian date — some number of days

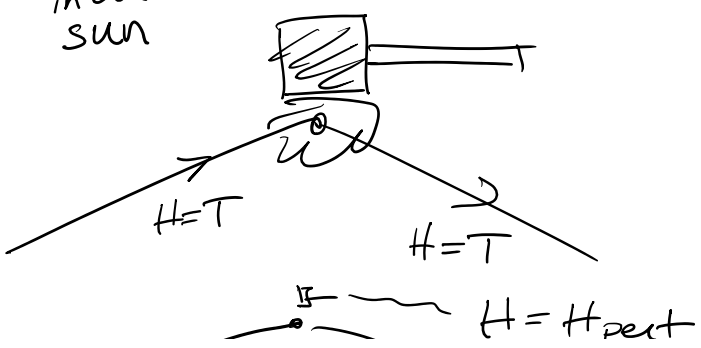
Integrate forward Δt = 4 days

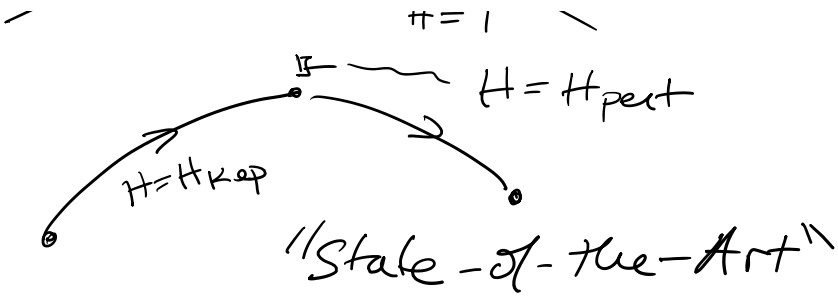
Decompose it into 2 parts

$H = H_{KEP} + \varepsilon H_{pert}$

H_{KEP}: $T + U$ { include sun

H_{pert}: { Potential Energy due to all planets only




n = 1
H = Hpert
H = HKep
"State-of-the-Art"