

if  $(\hat{L} \cdot \hat{N})$  large : Bright diffuse lighting  
 if  $(\hat{R} \cdot \hat{V})$  large : Bright specular reflection  
 if "metallic" then specular color = light source color  
 else specular color  $\cong$  material color

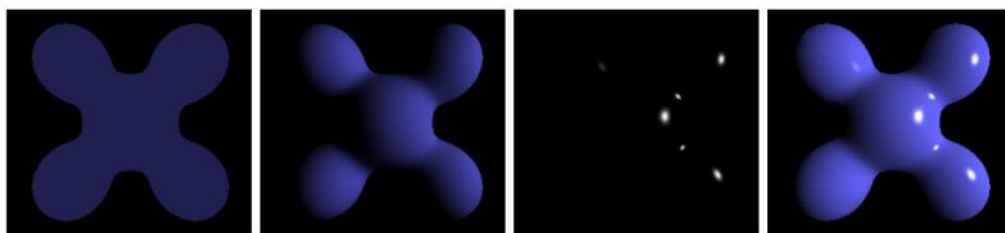
Light  $\underline{I}_m = (R, G, B)$  intensity of infalling light  
 $\underline{A} = (R, G, B)$  intensity of the ambient light

Note  $\underline{C} \underline{I} = \{C_r I_r, C_g I_g, C_b I_b\}$   
 component-wise multiplication

Object  $\underline{C} = (R, G, B)$  is the color of the object  
Geometry + Material  $\underline{S}$  is the specular highlight color, which is linearly interpolated according to  $(M_{sm})$

Material

- $m_a$ ,  $m_d$ ,  $m_s$  ambient diffuse specular
- $n$  - for transparent objects
- $m_{sp}$  - Phong exponent
- $m_{sm}$  is the "metalness" parameter of the material ( $0 \rightarrow$  very metallic)
- $\underline{C}$  color of the material

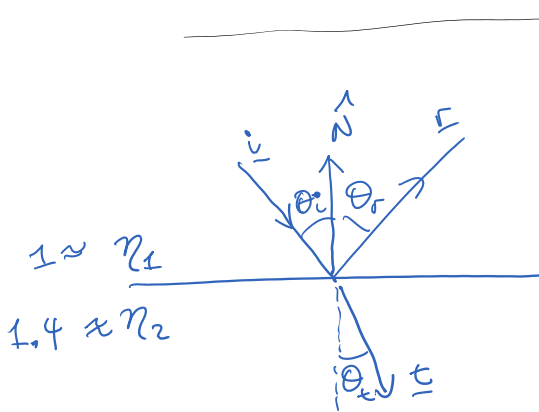


Ambient + Diffuse + Specular = Phong Reflection

$$\rho = m_a \underline{C} \underline{A} + \sum_{m \in \text{lights}} \left[ m_d \underline{C} \underline{I}_m \max(\hat{L}_m \cdot \hat{N}, 0) + m_s \underline{S} \underline{I}_m \max(\hat{R}_m \cdot \hat{V}, 0)^{m_{sp}} \right]$$

$$\underline{S} = m_{sm} \underline{C} + (1 - m_{sm})(1, 1, 1)$$

$$\hat{R}_m = 2(\hat{L}_m \cdot \hat{N})\hat{N} - \hat{L}_m$$



$$\theta_i = \theta_r$$

Reflection and Refraction

$$n_1 \sin \theta_i = n_2 \sin \theta_t$$

(Snell's Law)

$$\sin \theta_t = \frac{n_1}{n_2} \sin \theta_i$$

$$\sin \theta_i > \frac{n_2}{n_1} \Rightarrow \text{TIR}$$

total internal reflection

$$\underline{t} = \underline{t}_{||} + \underline{t}_{\perp}$$

FRESNEL EQUATIONS

$$T + R = 1$$

$$R_{\perp}(\theta_i) = \left( \frac{n_1 \cos \theta_i - n_2 \cos \theta_t}{n_1 \cos \theta_i + n_2 \cos \theta_t} \right)^2$$

$$R_{||}(\theta_i) = \left( \frac{n_2 \cos \theta_i - n_1 \cos \theta_t}{n_2 \cos \theta_i + n_1 \cos \theta_t} \right)^2$$

$$\sin^2 \theta_t = \left( \frac{n_1}{n_2} \right)^2 \sin^2 \theta_i = \left( \frac{n_1}{n_2} \right)^2 (1 - \cos^2 \theta_i)$$

$$\cos \theta_t = \sqrt{1 - \sin^2 \theta_t}$$

$$R(\theta_i) = \begin{cases} R_{\perp}(\theta_i) + R_{||}(\theta_i) & \text{for !TIR} \\ 1 & \text{for TIR} \end{cases}$$

$$T(\theta_i) = 1 - R(\theta_i)$$

Schlick's Approx (optimization)

$$R_{sch}(\theta_i) = R_0 + (1 - R_0)(1 - \cos \theta_i)^5$$

where  $R_0 = \left( \frac{n_1 - n_2}{n_1 + n_2} \right)^2$

careful:  $n_1 > n_2$

use  $\cos \Theta_t$  instead of  $\cos \Theta_i$

$$R_{\text{ref}}(\Theta_i) = \begin{cases} R_0 + (1-R_0)(1-\cos \Theta_i)^\xi & n_1 < n_2 \\ R_0 + (1-R_0)(1-\cos \Theta_t)^\xi & n_2 > n_1 \text{ and no TIR!} \\ 1 & \text{TIR} \end{cases}$$

$$\max(R_m \cdot \hat{V}, 0) = \max((1-\lambda)^{\beta\gamma}, 0) = \max((1-\lambda)^\beta, 0)^\gamma$$

where  $\lambda = 1 - R_m \cdot \hat{V}$  and  $\beta = \frac{\alpha}{\gamma}$  Small

$$\leadsto \approx \max(1 - \beta\lambda, 0)^\gamma$$

If we choose  $\gamma$  to be an integer power of 2, i.e.,  $\gamma = 2^n$

$(1-\beta\lambda)^\gamma \rightarrow$  square  $(1-\beta\lambda)$   $n$ -times

$$(1-\beta\lambda)^\gamma = (1-\beta\lambda)^{2^n} = (1-\beta\lambda)^{2 \cdot 2 \cdot 2 \dots}$$

holds for sufficiently large  $\gamma$

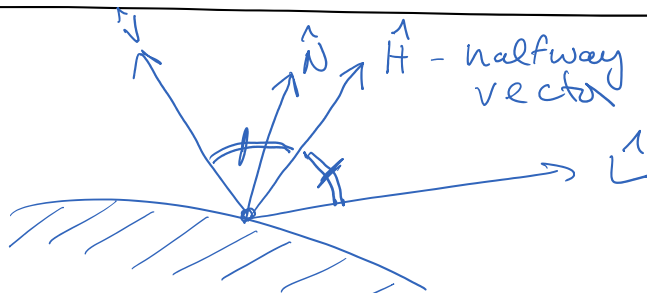
(4 or 8)

$$\lambda = (R_m \cdot \hat{V}) \cdot (R_m \cdot \hat{V}) / 2$$

or

$$= (R_m \times \hat{V}) \cdot (R_m \times \hat{V}) / 2$$

Blinn-Phong Model



$$(R_n \cdot \hat{V}) \longmapsto (\hat{N} \cdot \hat{H})^{\alpha'}$$

$\alpha' > \alpha$

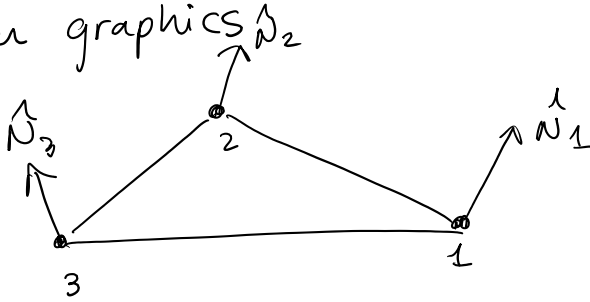
For a front lit surface  $\alpha' = 4\alpha$

$\hat{H}$  is independent of position if  $L \rightarrow \infty$

For a triangle  
 $\hat{H}$  is independent of position if  $L \rightarrow \infty$   
 $V \rightarrow \infty$   
↪ calculate this once.

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Raster graphics



(Gouraud)  
Shading